Date : 02.06.2013

CODE

9

IIT STUDY CIRCLE

A SHRIRAM GROUP INITIATIVE

Time : 3 hrs. **Max. Marks: 180**

Answers & Solutions
for
JEE (Advanced)-2013

PAPER - 1 (Code - 9)

INSTRUCTIONS

Question Paper Format

The question paper consists of three parts (Physics, Chemistry and Mathematics). Each part consists of three sections.

Section 1 contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

Section 2 contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

Section 3 contains **5 questions**. The answer to each question is a single-digit integer, ranging from 0 to 9 (both inclusive).

Marking Scheme

For each question in **Section 1**, you will be awarded **2 marks** if you darken the bubble corresponding to the correct answer and **zero mark** if no bubbles are darkened. **No negative** marks will be awarded for incorrect answers in this section.

For each question in **Section 2**, you will be awarded **4 marks** if you darken **all** the bubble(s) corresponding to only the correct answer(s) and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.

For each question in **Section 3**, you will be awarded **4 marks** if you darken the bubble corresponding to only the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus one (–1) mark** will be awarded.

PART–I : PHYSICS

SECTION - 1 : (Only One Option Correct Type)

This section contains **10 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

1. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30 mW and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is

   (A) 0.3×10^{-17} kg ms^{-1} (B) 1.0×10^{-17} kg ms^{-1}

   (C) 3.0×10^{-17} kg ms^{-1} (D) 9.0×10^{-17} kg ms^{-1}

Answer (B)

Hint : $p \quad \frac{E}{C} \quad \frac{P \ t}{C} \quad \frac{30 \ \ 10^{-3} \ \ 100 \ \ 10^{-9}}{3 \ \ 10^8} \quad 10^{-17}$ kg ms^{-1}

2. One end of a horizontal thick copper wire of length $2L$ and radius $2R$ is welded to an end of another horizontal thin copper wire of length L and radius R. When the arrangement is stretched by applying forces at two ends, the ratio of the elongation in the thin wire to that in the thick wire is

   (A) 0.25 (B) 0.50

   (C) 2.00 (D) 4.00

Answer (C)

Hint : Force will be same

Now, $\frac{FL}{AY}$

$\frac{1}{2} \quad \frac{L}{R^2} \quad \frac{(2R)^2}{2L} \quad 2$

3. The work done on a particle of mass m by a force, $K \ \frac{x}{(x^2 \quad y^2)^{3/2}} \hat{i} \quad \frac{y}{(x^2 \quad y^2)^{3/2}} \hat{j}$ (K being a constant of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the $x - y$ plane is

   (A) $\frac{2K}{a}$ (B) $\frac{K}{a}$

   (C) $\frac{K}{2a}$ (D) 0

Answer (D)

Hint : $F \quad k \ \frac{x\hat{i}}{(x^2 \quad y^2)^{3/2}} \quad \frac{y\hat{j}}{(x^2 \quad y^2)^{3/2}}$

$dW \quad F.dx$

$dW \quad k \ \frac{xdx \quad ydy}{(x^2 \quad y^2)^{3/2}}$

let $x^2 + y^2 = r^2$

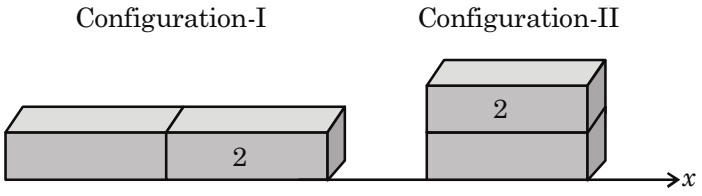
$xdx + ydy = r^2$

$$dW = \frac{krdr}{r^3} = \frac{k}{r^2}dr$$

$$W = \left[\frac{k}{r}\right]_{r_1}^{r_2}$$

Now, $r_1 = a$, $r_2 = a$ $W = 0$

4. Two rectangular blocks, having identical dimensions, can be arranged either in configuration-I or in configuration-II as shown in the figure. One of the blocks has thermal conductivity and the other 2 . The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration-I. The time to transport the same amount of heat in the configuration-II is



(A) 2.0 s (B) 3.0 s

(C) 4.5 s (D) 6.0 s

Answer (A)

Hint : $\frac{Q}{t} = \frac{T_1 - T_2}{\frac{l}{A} + \frac{l}{2A}}$ [in case (i)]

$$\frac{Q}{t'} = (T_1 - T_2)\frac{A}{l} + \frac{2A}{l}$$

$$\frac{t'}{t} = \frac{2}{9}$$

$t' = 2$ s

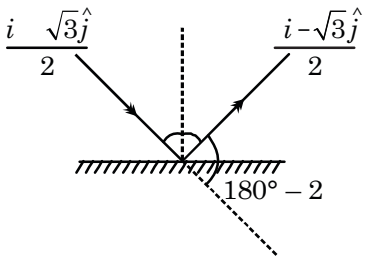
5. A ray of light travelling in the direction $\frac{1}{2}(\hat{i} + \sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i} - \sqrt{3}\hat{j})$. The angle of incidence is

(A) 30° (B) 45°

(C) 60° (D) 75°

Answer (A)

Hint : $\cos(180 - 2\ ) = \frac{\frac{i + \sqrt{3}\hat{j}}{2} \cdot \frac{i - \sqrt{3}\hat{j}}{2}}{\left|\frac{i + \sqrt{3}\hat{j}}{2}\right|\left|\frac{i - \sqrt{3}\hat{j}}{2}\right|}$



$$\cos 2 \quad \frac{\frac{(1 \quad 3)}{4}}{1}$$

$$\cos 2 \quad \frac{1}{2}$$

$$\cos 2 \quad \frac{1}{2}$$

$$2 \quad 60$$

$$= 30°$$

6. The diameter of a cylinder is measured using a Vernier callipers with no zero error. It is found that the zero of the Vernier scale lies between 5.10 cm and 5.15 cm of the main scale. The Vernier scale has 50 divisions equivalent to 2.45 cm. The 24^{th} division of the Vernier scale exactly coincides with one of the main scale divisions. The diameter of the cylinder is

(A) 5.112 cm (B) 5.124 cm

(C) 5.136 cm (D) 5.148 cm

Answer (B)

Hint : 1 MSD = 5.15 – 5.10 = 0.05 cm

$$1 \text{ VSD} = \frac{2.45}{50} = 0.049 \text{ cm}$$

LC = 1 MSD – 1 VSD

= 0.001 cm

Reading = 5.10 + L.C. × 24

= 5.10 + 0.024

= 5.124 cm

7. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is

(A) 1 : 4 (B) 1 : 2

(C) 6 : 9 (D) 8 : 9

Answer (D)

Hint : $P \quad \frac{RT}{M}$

$$\frac{P_1}{P_2} \quad \frac{1}{M_1} \quad \frac{M_2}{2}$$

$$\frac{4}{3} \quad \frac{1}{2}\frac{3}{2}$$

$$\frac{1}{2} \quad \frac{8}{9}$$

8. In the Young's double slit experiment using a monochromatic light of wavelength , the path difference (in terms of an integer n) corresponding to any point having half the peak intensity is

(A) $(2n \quad 1)\frac{}{2}$ (B) $(2n \quad 1)\frac{}{4}$

(C) $(2n \quad 1)\frac{}{8}$ (D) $(2n \quad 1)\frac{}{16}$

Answer (B)

Hint : $I \quad I_{\max}\cos^2 \quad \frac{}{2}$

$\frac{1}{2} \quad \cos^2 \quad \frac{}{2}$

$\cos \quad = 0$

$\frac{}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$

$x \quad \frac{}{4}, \frac{3}{4}, \frac{5}{4}$

$x \quad (2n \quad 1)\frac{}{4}$

9. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real and is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is

(A) 1 m (B) 2 m

(C) 3 m (D) 6 m

Answer (C)

Hint : $\frac{c}{} \quad \frac{f_{air}}{f_{med}} \quad \frac{3}{2}$

Now, $= +8$ m, $m \quad \frac{1}{3} \quad \frac{}{u} \quad u \quad 24$ m

$\frac{1}{f} \quad \frac{1}{} \quad \frac{1}{u}$

$\frac{1}{f} \quad \frac{1}{8} \quad \frac{1}{24} \quad \frac{4}{24}$

$f = 6$ m

$f \quad \frac{R}{1}$

$6 \quad \frac{R}{0.5} \quad R \quad 3$

10. A particle of mass m is projected from the ground with an initial speed u_0 at an angle with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is

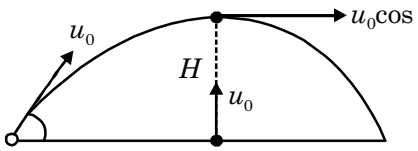
(A) $\frac{}{4}$ (B) $\frac{}{4}$

(C) $\frac{}{2}$ – (D) $\frac{}{2}$

Answer (A)

Hint : Speed of first particle at highest point = $u_0\cos$

Speed of second particle at highest point = $\sqrt{u_0^2 \quad 2gH}$



Now, $H \quad \frac{u_0^2 \sin^2}{2g}$

Speed of 2nd particle = $u_0\cos$

Final momentum = $mu_0 \cos \quad \hat{i} \quad mu_0 \cos \quad \hat{j}$

Angle = $\frac{}{4}$

SECTION - 2 : (One or More Options Correct Type)

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct**.

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11. A particle of mass M and positive charge Q, moving with a constant velocity $u_1 \quad 4\hat{i}\,\text{ms}^{-1}$, enters a region of uniform static magnetic field, normal to the x-y plane. The region of the magnetic field extends from $x = 0$ to $x = L$ for all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $u_2 \quad 2(\sqrt{3}\hat{i} \quad \hat{j})\text{ms}^{-1}$. The correct statement(s) is (are)

(A) The direction of the magnetic field is $-z$ direction

(B) The direction of the magnetic field is $+z$ direction

(C) The magnitude of the magnetic field $\frac{50\ M}{3Q}$ units

(D) The magnitude of the magnetic field is $\frac{100\ M}{3Q}$ units

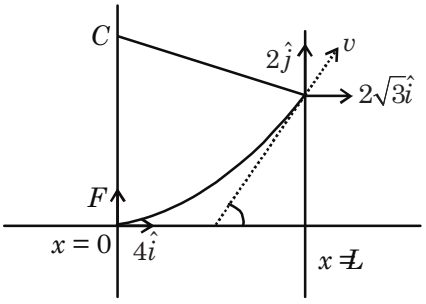
Answer (A, C)

Hint : $t \quad \frac{M}{qB}$

Clearly = 30° = $\frac{}{6}$

$B \quad \frac{M}{6q \quad 10 \quad 10^{-3}} \quad \frac{100\ M}{6q} \quad \frac{50\ M}{3Q}$

B must be in $-z$ direction.



12. Two non-conducting solid spheres of radii R and $2R$, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance $2R$ from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be

(A) -4 (B) $\frac{32}{25}$

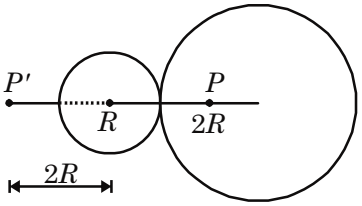
(C) $\frac{32}{25}$ (D) 4

Answer (B, D)

Hint : $E_P = 0$

$\frac{\rho_1 \frac{4}{3} R^3}{4\varepsilon_0 (2R)^2} \quad \frac{\rho_2 (R)}{3\varepsilon_0}$

$\frac{\rho_1}{\rho_2} \quad 4$



$E_{P'} \quad 0$

$\frac{\rho_1 \frac{4}{3} R^3}{4\varepsilon_0 (2R)^2} \quad \frac{\rho_2 \frac{4}{3} (2R)^3}{4\varepsilon_0 (5R)^2}$

$\frac{\rho_1}{\rho_2} \quad \frac{32}{25}$

13. A solid sphere of radius R and density ρ is attached to one end of a mass-less spring of force constant k. The other end of the spring is connected to another solid sphere of radius R and density 3ρ. The complete arrangement is placed in a liquid of density 2ρ and is allowed to reach equilibrium. The correct statement(s) is (are)

(A) The net elongation of the spring is $\frac{4\pi R^3 \rho g}{3k}$

(B) The net elongation of the spring is $\frac{8\pi R^3 \rho g}{3k}$

(C) The light sphere is partially submerged

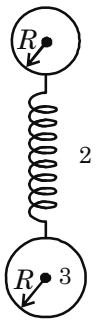
(D) The light sphere is completely submerged

Answer (A, D)

Hint : At equilibrium, for upper sphere

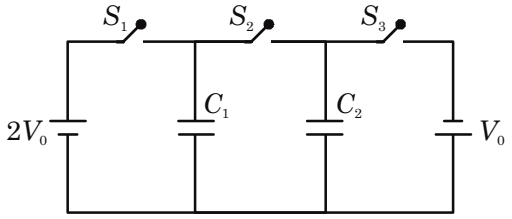
$kx \quad \frac{4}{3}\pi R^3 \rho g \quad \frac{4}{3}\pi R^3 (2\rho) g$

$x \quad \frac{4\pi R^3 \rho g}{3k}$



The system is completely submerged as total weight = Total Buoyant force.

14. In the circuit shown in the figure, there are two parallel plate capacitors each of capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed. After some time,



(A) The charge on the upper plate of C_1 is $2CV_0$

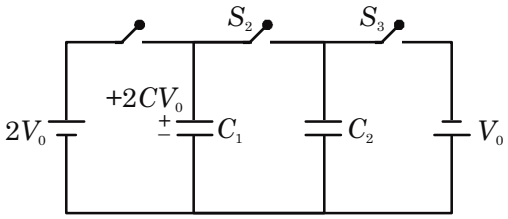
(B) The charge on the upper plate of C_1 is CV_0

(C) The charge on the upper plate of C_2 is 0

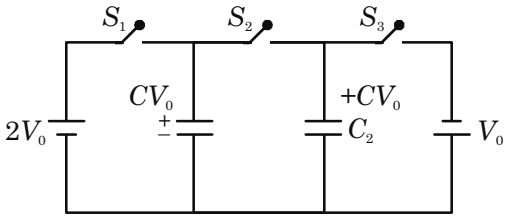
(D) The charge on the upper plate of C_2 is $-CV_0$

Answer (B, D)

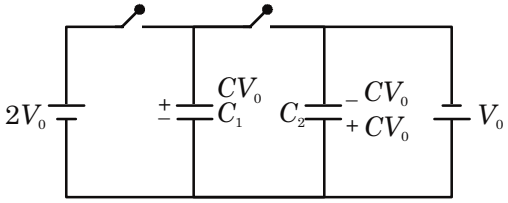
Hint : When S_1 is pressed and released



When S_2 is pressed and released



When S_3 is pressed



15. A horizontal stretched string, fixed at two ends, is vibrating in its fifth harmonic according to the equation, $y(x, t) = (0.01\text{ m}) \sin[(62.8\text{ m}^{-1})x] \cos[(628\text{ s}^{-1})t]$. Assuming = 3.14, the correct statement(s) is (are)

(A) The number of nodes is 5

(B) The length of the string is 0.25 m

(C) The maximum displacement of the midpoint of the string, from its equilibrium position is 0.01 m

(D) The fundamental frequency is 100 Hz

Answer (B, C)

Hint : No. of nodes = 6, $\frac{2}{k} = \frac{2 \quad 3.14}{62.8} \quad 0.1\text{ m}$

Length = $\frac{5}{2} \quad 0.25\text{ m}$

The mid-point is antinode. Its maximum displacement = 0.01 m

$f = \frac{v}{2l} \quad \frac{}{k \quad 2l} \quad 20\text{ Hz}$

SECTION - 3 : (Integer Value Correct Type)

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (*both inclusive*).

16. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is

Answer (5)

Hint : As power is constant, $P \times t = kE$

$$0.5 \quad 5 \quad \frac{1}{2} \quad (0.2)v^2 \quad 0$$

$$2.5 = 0.1\ v^2$$

$$v = 5 \text{ m/s}$$

17. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is

Answer (8)

Hint : By conservation of angular momentum,

$$\frac{1}{2} \quad 50 \quad (0.4)^2 \quad 10 \quad \frac{1}{2} \quad 50 \quad (0.4)^2 \quad 2 \quad 2 \quad 6.25 \quad (0.2)^2$$

$$\frac{40}{4 \quad 1} \quad 8 \text{ rad/s}$$

18. A freshly prepared sample of a radioisotope of half-life 1386 s has activity 10 disintegrations per second. Given that $ln\ 2 = 0.693$, the fraction of the initial number of nuclei (expressed in nearest integer percentage) that will decay in the first 80 s after preparation of the sample is

Answer (4)

Hint : $N \quad N_0 e^{\ t}$

$$\frac{N}{N_0} \quad e^{\ t}$$

$$\frac{N}{N_0} \quad e^{\frac{\ln 2}{1386} 80}$$

$$\frac{N}{N_0} \quad e^{\frac{0.693 \ 80}{1386}}$$

$$\frac{N}{N_0} \quad e^{0.04}$$

$$\frac{N}{N_0} \quad \frac{1}{e}^{0.04}$$

Fraction of nuclei decayed = $1 \quad \frac{N}{N_0} \quad 1 \quad \frac{1}{e}^{0.04} = 0.04 = 4\%$

19. A bob of mass m, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is

Answer (5)

Hint : Speed of first bob at highest point = $\sqrt{gl_1}$

For elastic collision between objects of same mass, velocities are exchanged speed of 2nd bob = $\sqrt{gl_1}$

but $\sqrt{gl_1}$ $\sqrt{5gl_2}$

$\frac{l_1}{l_2}$ 5

20. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is

Answer (1)

Hint : V $\frac{hf}{e}$ $\frac{}{e}$

Slope = $\frac{h}{e}$.

It is same for both.

PART–II : CHEMISTRY

SECTION - 1 : (Only One Option Correct Type)

This section contains **10 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

21. Upon treatment with ammoniacal H_2S, the metal ion that precipitates as a sulfide is

(A) Fe(III) (B) Al(III)

(C) Mg(II) (D) Zn(II)

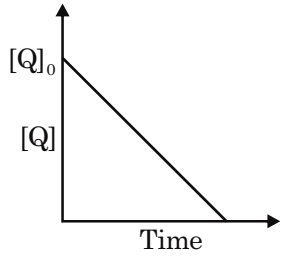
Answer (D)

Hint : Upon treatment with ammoniacal H_2S, Zn^{2+} ion gets precipitated as ZnS. Fe^{3+} ion and Al^{3+} ions also get precipitated as hydroxides but not as sulphide.

22. In the reaction,

$P + Q \longrightarrow R + S$

The time taken for 75% reaction of P is twice the time taken for 50% reaction of P. The concentration of Q varies with reaction time as shown in the figure. The overall order of the reaction is



(A) 2 (B) 3

(C) 0 (D) 1

Answer (D)

Hint : The order of reaction with respect to P is one since $t_{3/4}$ is twice of $t_{1/2}$. From the given graph the order of reaction with respect to Q is zero. Therefore, overall order of reaction is one.

23. Consider the following complex ions, P, Q and R.

$P = [FeF_6]^{3-}$, $Q = [V(H_2O)_6]^{2+}$ and $R = [Fe(H_2O)_6]^{2+}$

The correct order of the complex ions, according to their spin-only magnetic moment values (in B.M.) is

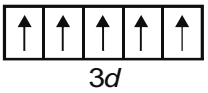
(A) R < Q < P (B) Q < R < P

(C) R < P < Q (D) Q < P < R

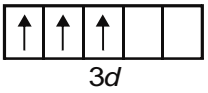
Answer (B)

Hint : The electronic configuration of central metal ion in complex ions P, Q and R are

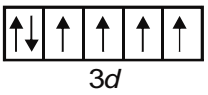
P $[FeF_6]^3$; Fe^3 :



$Q = [V(H_2O)_6]^{2+}$; V^{2+}:

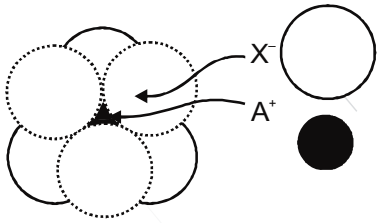


$R = [Fe(H_2O)_6]^{2+}$; Fe^{2+}



The correct order of spin only magnetic moment is Q < R < P

24. The arrangement of X^- ions around A^+ ion in solid AX is given in the figure (not drawn to scale). If the radius of X^- is 250 pm, the radius of A^+ is



(A) 104 pm (B) 125 pm

(C) 183 pm (D) 57 pm

Answer (A)

Hint : Cation A^+ occupies octahedral void formed by anions X^-. The maximum radius ratio for a cation to accommodate a octahedral void without distortion is 0.414. Radius of anion X^- is 250 pm.

$$\frac{R_A}{R_X} \quad 0.414$$

$$R_A \quad 0.414 \quad 250 \quad 103.50 \quad 104 \text{ pm}$$

25. Concentrated nitric acid, upon long standing, turns yellow-brown due to the formation of

(A) NO (B) NO_2

(C) N_2O (D) N_2O_4

Answer (B)

Hint : Conc. HNO_3 slowly decomposes as

$4HNO_3 \quad 4NO_2 + 2H_2O + O_2$

It acquires yellow-brown colour due to the formation of NO_2.

26. The compound that does **NOT** liberate CO_2, on treatment with aqueous sodium bicarbonate solution, is

(A) Benzoic acid
(B) Benzenesulphonic acid
(C) Salicylic acid
(D) Carbolic acid (Phenol)

Answer (D)

Hint : Carbolic acid (Phenol) is weaker acid than carbonic acid and hence does not liberate CO_2 on treatment with aq. $NaHCO_3$ solution. Benzoic acid, benzenesulphonic acid and salicylic are more acidic than carbonic acid and hence will liberate CO_2 with aq. $NaHCO_3$ solution.

27. Sulfide ores are common for the metals

(A) Ag, Cu and Pb
(B) Ag, Cu and Sn
(C) Ag, Mg and Pb
(D) Al, Cu and Pb

Answer (A)

Hint : Silver, copper and lead are commonly found in earth's crust as Ag_2S (silver glance), $CuFeS_2$ (Copper pyrites) and PbS (Galena)

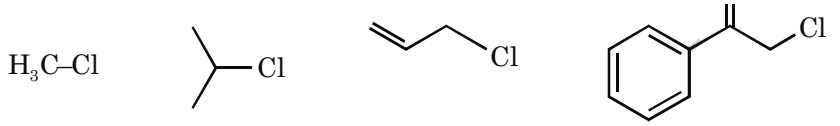
28. Methylene blue, from its aqueous solution, is adsorbed on activated charcoal at 25°C. For this process, the correct statement is

(A) The adsorption requires activation at 25°C
(B) The adsorption is accompanied by a decrease in enthalpy
(C) The adsorption increases with increase of temperature
(D) The adsorption is irreversible

Answer (B)

Hint : The adsorption of methylene blue on activated charcoal is physiosorption which is exothermic, multilayer and does not have energy barrier.

29. KI in acetone, undergoes S_N2 reaction with each of P, Q, R and S. The rates of the reaction vary as

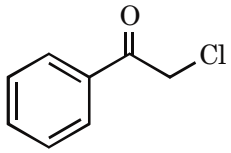
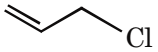
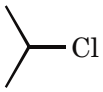


P Q R S

(A) P > Q > R > S
(B) S > P > R > Q
(C) P > R > Q > S
(D) R > P > S > Q

Answer (B)

Hint :

Compounds  : CH_3–Cl :  : 

S P R Q

Relative reactivities towards S_{N^2} reaction	1,00,000	:	200	:	79	:	0.02

30. The standard enthalpies of formation of CO_2(g), H_2O(l) and glucose(s) at 25°C are –400 kJ/mol, –300 kJ/mol and –1300 kJ/mol, respectively. The standard enthalpy of combustion per gram of glucose at 25°C is

(A) +2900 kJ
(B) –2900 kJ
(C) –16.11 kJ
(D) +16.11 kJ

Answer (C)

Hint : $C_6H_{12}O_6(s) + 6O_2(g) \quad 6CO_2(g) + 6H_2O(l)$

$H° = 6(-400) + 6(-300) - (-1300)$

$H° = -2900$ kJ/mol

$H \quad \frac{2900}{180} \quad 16.11$ kJ / gm

SECTION - 2 : (One or More Options Correct Type)

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct**.

---

31. The hyperconjugative stabilities of tert-butyl cation and 2-butene, respectively, are due to

(A) p (empty) and electron delocalisations

(B) * and electron delocalisations

(C) p (filled) and electron delocalisations

(D) p (filled) and electron delocalisations

Answer (A)

Hint : In hyperconjugation p (empty) electron delocalization for tert-butyl carbocation and * electron delocalization for 2-butene will take place.

32. Benzene and naphthalene form an ideal solution at room temperature. For this process, the true statement(s) is(are)

(A) G is positive

(B) S_{system} is positive

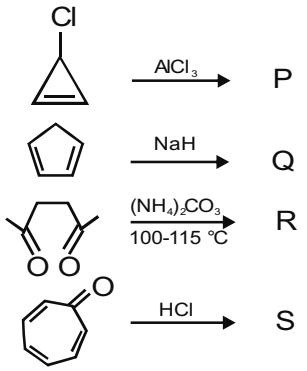
(C) $S_{surroundings} = 0$

(D) $H = 0$

Answer (B, C, D)

Hint : Benzene and naphthalene form an ideal solution. For an ideal solution, $H = 0$, $S_{system} > 0$ and $S_{surroundings} = 0$ because there is no exchange of heat energy between system and surroundings.

33. Among P, Q, R and S, the aromatic compound(s) is/are



(A) P

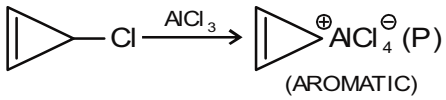
(B) Q

(C) R

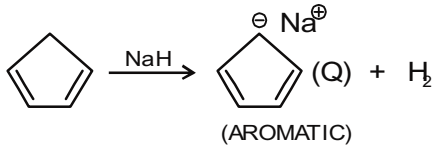
(D) S

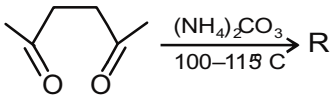
Answer (A, B, C, D)

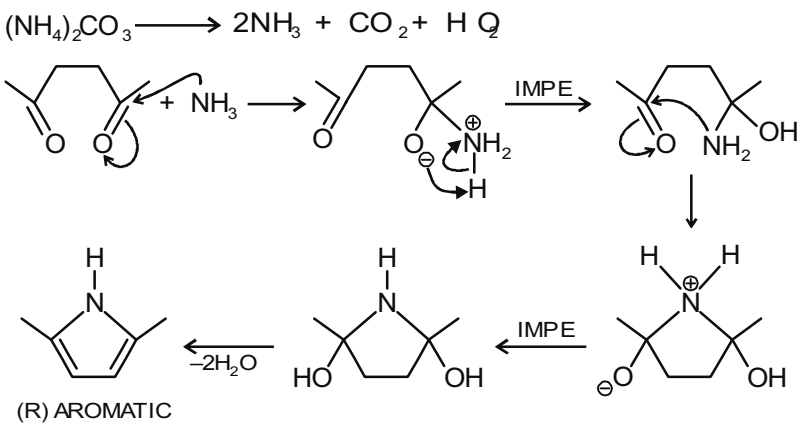
Hint : (i)

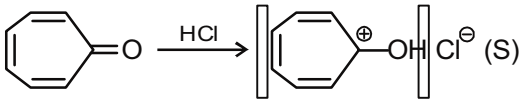


(ii)



(iii) 



(iv) 

34. The initial rate of hydrolysis of methyl acetate (1M) by a weak acid (HA, 1M) is $1/100^{th}$ of that of a strong acid (HX, 1M), at 25°C. The K_a of HA is

(A) 1×10^{-4} (B) 1×10^{-5}

(C) 1×10^{-6} (D) 1×10^{-3}

Answer (A)

Hint : Rate with respect to weak acid

$$R_1 = K[H^+]_{WA}[ester]$$

and rate with respect to strong acid

$$R_2 = K[H^+]_{SA}[ester]$$

$$\frac{R_1}{R_2} \quad \frac{[H\ ]_{WA}}{[H\ ]_{SA}} \quad \frac{1}{100}$$

$$H_{WA} \quad \frac{[H\ ]_{SA}}{100} \quad \frac{1}{100} \quad 0.01\,M \quad C$$

$$= 0.01$$

K_a for weak acid $= C^{\ 2}$

$$= 1(0.01)^2$$

$$= 1 \times 10^{-4}$$

35. The pair(s) of coordination complexes/ions exhibiting the same kind of isomerism is(are)

(A) $[Cr(NH_3)_5Cl]Cl_2$ and $[Cr(NH_3)_4Cl_2]Cl$ (B) $[Co(NH_3)_4Cl_2]^+$ and $[Pt(NH_3)_2(H_2O)Cl]^+$

(C) $[CoBr_2Cl_2]^{2-}$ and $[PtBr_2Cl_2]^{2-}$ (D) $[Pt(NH_3)_3](NO_3)Cl$ and $[Pt(NH_3)_3Cl]Br$

Answer (B, D)

Hint : The pair of complex ions $[Co(NH_3)_4Cl_2]^+$ and $[Pt(NH_3)_2(H_2O)Cl]^+$ show geometrical isomerism. The pair of complexes $[Pt(NH_3)_3(NO_3)]Cl$ and $[Pt(NH_3)_3Cl]Br$ show ionisation isomerism. The other pairs given do not have same type of isomerism.

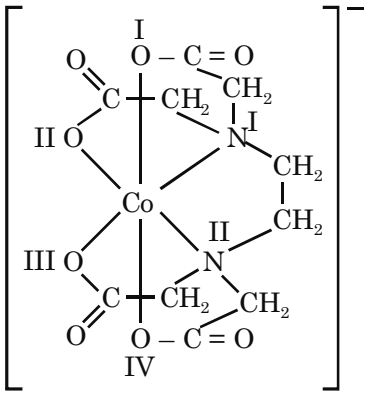
SECTION - 3 : (Integer Value Correct Type)

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (*both inclusive*).

36. $EDTA^{4-}$ is ethylenediaminetetraacetate ion. The total number of N—Co—O bond angles in $[Co(EDTA)]^{1-}$ complex ion is

Answer (8)

Hint :



So, bond angles are

(1) $N^{I}\,CoO^{I}$

(2) $N^{I}\,CoO^{II}$

(3) $N^{I}\,CoO^{III}$

(4) $N^{I}\,CoO^{IV}$

(5) $N^{II}\,CoO^{I}$

(6) $N^{II}\,CoO^{II}$

(7) $N^{II}\,CoO^{III}$

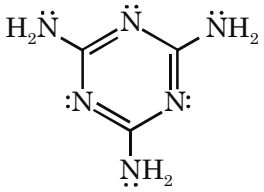
(8) $N^{II}\,CoO^{IV}$

So, total asked bond angles are 8.

37. The total number of lone-pairs of electrons in melamine is

Answer (6)

Hint : Structure of melamine is



So, melamine has six lone pair of electrons.

38. A tetrapeptide has —COOH group on alanine. This produces glycine (Gly), valine (Val), phenyl alanine (Phe) and alanine (Ala), on complete hydrolysis. For this tetrapeptide, the number of possible sequences (primary structures) with $—NH_2$ group attached to a chiral center is

Answer (4)

Hint : According to question C – Terminal must be alanine and N – Terminal do have chiral carbon means its should not be glycine so possible sequence is :

Val Phe Gly Ala

Val Gly Phe Ala

Phe Val Gly Ala

Phe Gly Val Ala

So, answer is (4).

39. The atomic masses of 'He' and 'Ne' are 4 and 20 a.m.u., respectively. The value of the de Broglie wavelength of 'He' gas at –73°C is "M" times that of the de Broglie wavelength of 'Ne' at 727°C 'M' is

Answer (5)

Hint : $\frac{He}{Ne}$ $\frac{M_{Ne}V_{Ne}}{M_{He}V_{He}}$

$$\frac{M_{Ne}\sqrt{\frac{3RT_{Ne}}{M_{Ne}}}}{M_{He}\sqrt{\frac{3RT_{He}}{M_{He}}}}$$

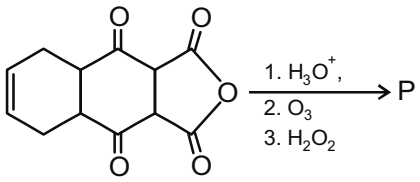
$$\frac{M_{Ne}\sqrt{\frac{T_{Ne}}{M_{Ne}}}}{M_{He}\sqrt{\frac{T_{He}}{M_{He}}}}$$

$$\sqrt{\frac{M_{Ne}T_{Ne}}{M_{He}T_{He}}}$$

$$\sqrt{\frac{205 \quad 1000}{4 \quad 200}}$$

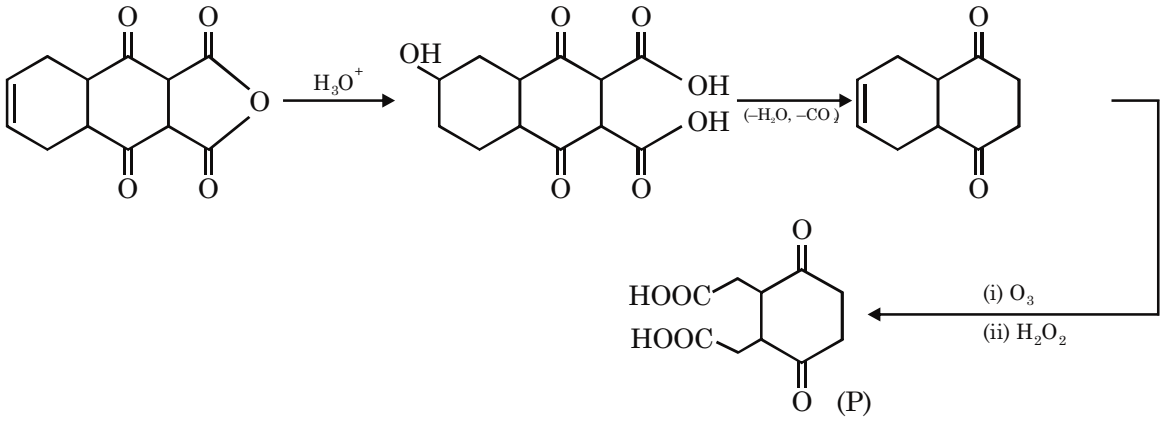
$_{He}$ 5 $_{Ne}$

40. The total number of carboxylic acid groups is the product P is



Answer (2)

Hint :



So, final product (P) has two carboxylic acid

Heating causes evaporation of water and due to which concentration of acid increases and in concentrated acidic medium decarboxylation as well as dehydration will take place.

PART–III : MATHEMATICS

SECTION - 1 : (Only One Option Correct Type)

This section contains **10 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

41. A curve passes through the point $1, \frac{}{6}$. Let the slope of the curve at each point (x, y) be $\frac{y}{x}$ $\sec \frac{y}{x}$, x 0 . Then the equation of the curve is

(A) $\sin \frac{y}{x}$ $\log x$ $\frac{1}{2}$

(B) $\operatorname{cosec} \frac{y}{x}$ $\log x$ 2

(C) $\sec \frac{2y}{x}$ $\log x$ 2

(D) $\cos \frac{2y}{x}$ $\log x$ $\frac{1}{2}$

Answer (A)

Hint : $\frac{dy}{dx}$ $\frac{y}{x}$ $\sec \frac{y}{x}$

Put $y = vx$

v $x\frac{dv}{dx}$ v $\sec v$

$\cos v\, dv$ $\frac{dx}{x}$

$\sin v = \ln x + c$

It passes through $1, \frac{}{6}$

$\frac{1}{2}$ c

$\sin \frac{y}{x}$ $\ln x$ $\frac{1}{2}$

42. Perpendiculars are drawn from points on the line $\frac{x \quad 2}{2}$ $\frac{y \quad 1}{1}$ $\frac{z}{3}$ to the plane $x + y + z = 3$. The feet of perpendiculars lie on the line

(A) $\frac{x}{5}$ $\frac{y \quad 1}{8}$ $\frac{z \quad 2}{13}$

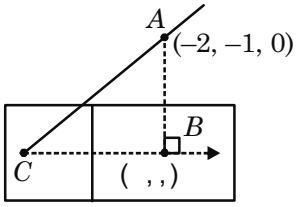
(B) $\frac{x}{2}$ $\frac{y \quad 1}{3}$ $\frac{z \quad 2}{5}$

(C) $\frac{x}{4}$ $\frac{y \quad 1}{3}$ $\frac{z \quad 2}{7}$

(D) $\frac{x}{2}$ $\frac{y \quad 1}{7}$ $\frac{z \quad 2}{5}$

Answer (D)

Hint : $\frac{x \quad 2}{2} \quad \frac{y \quad 1}{1} \quad \frac{z}{3} \quad , \qquad x \quad y \quad z \quad 3$



Any point on this line is $(2 \quad -2, - \quad -1, 3 \ )$

This point will satisfy the plane

$(2 \quad -2) + (- \quad -1) + (3 \ ) = 3$

$4 \quad -3 = 3$

$\frac{3}{2}$

So, point of intersection of plane is $1, \frac{5}{2}, \frac{9}{2} \quad C$.

Now, point on line is (–2, –1, 0) and direction ratio of AB (from figure) is $\frac{2}{1} \quad \frac{1}{1} \quad \frac{}{1} \quad k$.

Any general point on line AB is $(k - 2, k - 1, k)$.

This will satisfy equation so, $(k - 2) + (k - 1) + k = 3$.

$k = 2$

Therefore, $(\ , \ , \ ) \quad (0, 1, 2)$.

So, equation of line passing through BC is

$$\frac{x}{1} \quad \frac{y \quad 1}{\frac{7}{2}} \quad \frac{z \quad 2}{\frac{5}{2}}$$

$$\frac{x}{2} \quad \frac{y \quad 1}{7} \quad \frac{z \quad 2}{5}$$

43. For $a > b > c > 0$, the distance between (1, 1) and the point of intersection of the lines $ax \quad by \quad c \quad 0$ and $bx \quad ay \quad c \quad 0$ is less than $2\sqrt{2}$. Then

(A) $a + b - c > 0$

(B) $a - b + c < 0$

(C) $a - b + c > 0$

(D) $a + b - c < 0$

Answer (A)

Hint : $ax \quad by \quad c \quad 0 \qquad \ldots (1)$

$bx \quad ay \quad c \quad 0 \qquad \ldots (2)$

Solving, $x \quad \frac{c}{a \quad b}$

Also from (1) & (2)

$y = x$

Point of intersection lies on $y = x$

$y \quad \frac{c}{a \quad b}$

Given, $\sqrt{1 \quad \frac{c}{a \quad b}^{2} \quad 1 \quad \frac{c}{a \quad b}^{2}} \quad 2\sqrt{2}$

$\sqrt{2} \quad 1 \quad \frac{c}{a \quad b} \quad 2\sqrt{2}$

$\frac{a \quad b \quad c}{a \quad b} \quad 2$

$a \quad b \quad c \quad 2a \quad 2b$

$a \quad b \quad c \quad 0$

44. The area enclosed by the curves $y \quad \sin x \quad \cos x$ and $y \quad |\cos x \quad \sin x|$ over the interval $0, \frac{}{2}$ is

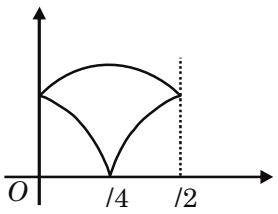
(A) $4(\sqrt{2} \quad 1)$ (B) $2\sqrt{2}(\sqrt{2} \quad 1)$

(C) $2(\sqrt{2} \quad 1)$ (D) $2\sqrt{2}(\sqrt{2} \quad 1)$

Answer (B)

Hint : $\int_{0}^{/2} (\sin x \quad \cos x)dx \quad \int_{0}^{/4} (\cos x \quad \sin x)dx \quad \int_{/4}^{/2} (\sin x \quad \cos x)dx$

$= \quad |\cos x|_{0}^{/2} \quad |\sin x|_{0}^{/2} \quad |\sin x|_{0}^{/4} \quad |\cos x|_{0}^{/4} \quad |\cos x|_{/4}^{/2} \quad |\sin x|_{/4}^{/2}$



$= \quad (0 \quad 1) \quad (1 \quad 0) \quad \frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}} \quad 1 \quad 0 \quad \frac{1}{\sqrt{2}} \quad 1 \quad \frac{1}{\sqrt{2}}$

$= 2 \quad \sqrt{2} \quad 1 \quad \frac{1}{\sqrt{2}} \quad 1 \quad \frac{1}{\sqrt{2}}$

$= 2 \quad 2\sqrt{2} \quad 2$

$= 4 \quad 2\sqrt{2}$

$= 2\sqrt{2}(\sqrt{2} \quad 1)$

45. Four persons independently solve a certain problem correctly with probabilities $\frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}$. Then the probability that the problem is solved correctly by at least one of them is

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$

(C) $\frac{3}{256}$ (D) $\frac{253}{256}$

Answer (A)

Hint : $P(A) \quad \frac{1}{2}, P(B) \quad \frac{3}{4}, P(C) \quad \frac{1}{4}, P(D) \quad \frac{1}{8}$

$P(A \quad B \quad C \quad D) = 1 \quad \overline{P(A \quad B \quad C \quad D)}$

$= 1 \quad P(\bar{A} \quad \bar{B} \quad \bar{C} \quad \bar{D})$

$= 1 \quad P(\bar{A})\, P(\bar{B})\, P(\bar{C})\, P(\bar{D})$

$= 1 \quad \frac{1}{2} \quad \frac{1}{4} \quad \frac{3}{4} \quad \frac{7}{8}$

$= 1 \quad \frac{21}{256}$

$= \frac{235}{256}$

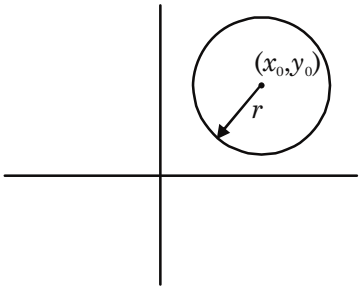
46. Let complex numbers and $\frac{1}{=}$ lie on circles $(x \quad x_0)^2 \quad (y \quad y_0)^2 \quad r^2$ and $(x \quad x_0)^2 \quad (y \quad y_0)^2 \quad 4r^2$, respectively. If $z_0 \quad x_0 \quad iy_0$ satisfies the equation $2|z_0|^2 \quad r^2 \quad 2$, then $| \quad |$

(A) $\frac{1}{\sqrt{2}}$ (B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{7}}$ (D) $\frac{1}{3}$

Answer (C)

Hint : lies on $(x \quad x_0)^2 \quad (y \quad y_0)^2 \quad r^2$ ... (1)



$\frac{1}{-} \quad \frac{}{|\ |^2}$ lies on $(x \quad x_0)^2 \quad (y \quad y_0)^2 \quad 4r^2$ ... (2)

Let be $_1 + i \ _1$

lies on $(x^2 \quad y^2) \quad (x_0^2 \quad y_0^2) \quad 2(xx_0 \quad yy_0) \quad r^2$

$|\ |^2 \quad |z_0|^2 \quad 2(\ _1x_0 \quad \ _1y_0) \quad r^2$ ... (3)

From eq. (2), $\frac{1}{|\ |^2}\ |z_0|^2\ \ 2\frac{(\ _1x_0\ \ \ _1y_0)}{|\ |^2}\ \ 4r^2$

$1\ \ |\ |^2|z_0|^2\ \ 2(\ _1x_0\ \ \ _1y_0)\ \ 4r^2|\ |^2$ ... (4)

From (3) – (4), $|\ |^2\ \ 1\ \ |z_0|^2(1\ \ |\ |^2)\ \ r^2\ 1\ \ 4|\ |^2$

$(|\ |^2\ \ 1)(1\ \ |z_0|^2)\ \ r^2\ 1\ \ 4|\ |^2$ ...(5)

Now, $r^2\ \ 2\ |z_0|^2\ \ 1$

$\frac{r^2}{|z_0|^2\ \ 1}\ \ 2$

Substituting in (5),

$|\ |^2\ \ 1\ \ \ 2(1\ \ 4|\ |^2)$

$|\ |^2\ \ 1\ \ \ 2\ \ 8|\ |^2$

$7|\ |^2\ \ 1$

$|\ |\ \ \frac{1}{\sqrt{7}}$

47. The number of points in $(-\ ,\ )$, for which $x^2\ \ x\sin x\ \ \cos x\ \ 0$, is

(A) 6 (B) 4

(C) 2 (D) 0

Answer (C)

Hint : $f(x)\ \ x^2\ \ x\sin x\ \ \cos x$

$f\,(x)\ \ 2x\ \ x\cos x\ \ \sin x\ \ \sin x$

$= x(2 - \cos x)$

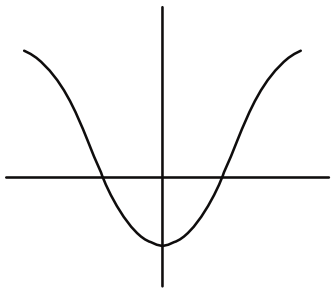
$f(x)$ is increasing $x > 0$

$f(x)$ is decreasing $x < 0$

$f(0) = -1$

$f(\ )$

$f(-\ )$



48. Let $f:\ \frac{1}{2}, 1$ (the set of all real numbers) be a positive, non-constant and differentiable function such that $f(x) < 2f(x)$ and $f\ \frac{1}{2}\ \ 1$. Then the value of $\int_{1/2}^{1} f(x)\,dx$ lies in the interval

(A) $(2e - 1,\ 2e)$ (B) $(e - 1,\ 2e - 1)$

(C) $\frac{e\ \ 1}{2}, e\ \ 1$ (D) $0, \frac{e\ \ 1}{2}$

Answer (D)

Hint : $\frac{dy}{dx} \quad 2y$

$e^{\ 2x}\frac{dy}{dx} \quad 2ye^{\ 2x}$

$\frac{d}{dx}(ye^{\ 2x}) \quad 0$

ye^{-2x} is decreasing function

As, $\frac{1}{2} \quad x \quad 1$

$e^{-1} > ye^{-2x} > y(1)e^{-2}$

$e^{2}x^{-1} > y > y(1)e^{2}x^{-2}$

$$\int_{1/2}^{1} e^{2x\ 1}dx \quad \int_{1/2}^{1} ydx \quad \int_{1/2}^{1} y(1)e^{2x\ 2} \quad 0$$

$$0 \quad \int_{1/2}^{1} ydx \quad \frac{e \quad 1}{2}$$

49. Let $PR \quad 3\hat{i} \quad \hat{j} \quad 2\hat{k}$ and $SQ \quad \hat{i} \quad 3\hat{j} \quad 4\hat{k}$ determine diagonals of a parallelogram $PQRS$ and $PT \quad \hat{i} \quad 2\hat{j} \quad 3\hat{k}$ be another vector. Then the volume of the parallelepiped determined by the vectors PT, PQ and PS is

(A) 5 (B) 20

(C) 10 (D) 30

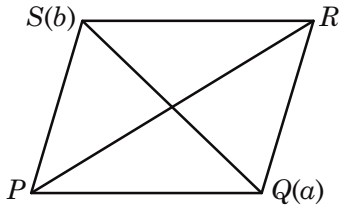
Answer (C)

Hint : $PR \quad a \quad b \quad 3\hat{i} \quad \hat{j} \quad 2\hat{k} \quad d_1$

$SQ \quad a \quad b \quad \hat{i} \quad 3\hat{j} \quad 4\hat{k} \quad d_2$

$a \quad 2\hat{i} \quad \hat{j} \quad 3\hat{k}$

$b \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$



Volume of the required parallelopiped $= \begin{vmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

$= |2(6-2) + 1(3-1) - 3(2-2)|$

$= 10$ cubic units

OR

Area of parallelogram $= \frac{1}{2}|d_1 \quad d_2|$

Volume of parallelepiped $= \frac{1}{2}\begin{Vmatrix} 3 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & 2 & 3 \end{Vmatrix} = 10$ cubic units.

50. The value of $\cot \sum_{n\ 1}^{23} \cot^{1} 1 \sum_{k\ 1}^{n} 2k$ is

(A) $\frac{23}{25}$ (B) $\frac{25}{23}$

(C) $\frac{23}{24}$ (D) $\frac{24}{23}$

Answer (B)

Hint : $\sum_{n\ 1}^{23} \cot^{1} 1 \quad k(k \quad 1)$

$= \sum_{n\ 1}^{23} \tan^{1}(k \quad 1) \quad \tan^{1}(k)$

$= \tan^{-1}(24) - \tan^{-1}(1)$

Now, $\cot (\tan^{1}(24) \quad \tan^{1}(1)$

$= \cot \tan^{1} \frac{24 \quad 1}{1 \quad 24 \ 1}$

$= \cot \tan^{1} \frac{23}{25}$

$= \frac{25}{23}$

SECTION - 2 : (One or More Options Correct Type)

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct**.

51. Let $f(x) = x \sin \quad x$, $x > 0$. Then for all natural numbers n, $f(x\ )$ vanishes at

(A) A unique point in the interval $n, n \quad \frac{1}{2}$

(B) A unique point in the interval $n \quad \frac{1}{2}, n \quad 1$

(C) A unique point in the interval $(n, n + 1)$

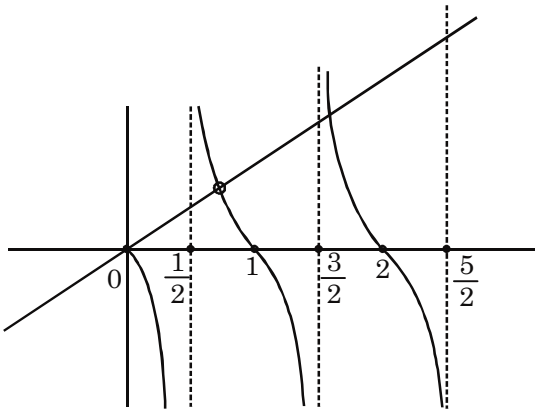
(D) Two points in the interval $(n, n + 1)$

Answer (B, C)

Hint : $f(x) = x \sin \ x$

$f(x\ ) = \sin \quad x + \quad x \cos \quad x = 0$

$-\tan x = x$



Clear $f(x)$ has one root in $n \frac{1}{2}, n 1$, also $f(x)$ has one root in $(n, n+1)$.

52. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. Then the lengths of the sides of the rectangular sheet are

(A) 24 (B) 32

(C) 45 (D) 60

Answer (A, C)

Hint :

$V (8 2x)(15 2x)x$

$= 4x^3 46 x^2 120 {}^2x$

$\frac{dV}{dx} 12x^2 92 x 120 {}^2 0$, at $x = 5$

$60 {}^2 230 150 0$

$6 {}^2 23 15 0$

$(6 5)(3) 0$

For $= 3$ lengths of sides are 45, 24

53. For 3×3 matrices M and N, which of the following statement(s) is (are) **NOT** correct?

(A) N^TMN is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(B) $MN - NM$ is skew symmetric for all symmetric matrices M and N

(C) MN is symmetric for all symmetric matrices M and N

(D) $(adj\ M)\ (adj\ N) = adj(MN)$ for all invertible matrices M and N

Answer (C, D)

Hint :

$(N^TMN)^T = - N^TM^T(N^T)^T$

N^TM^TN

(A) If M is skew symmetric, $(N^TMN)^T = -N^TMN$,

Hence skew symmetric.

If M is symmetric, $(M^TMN)^T = N^TMN$,

Hence symmetric.

Option (A) is correct

(B) $(MN - NM)^T = (MN)^T - (NM)^T$

$= N^TM^T - M^TN^T$

$= -(M^TM^T - N^TM^T)$

$= -(MN - NM)$

Skew symmetric, option (B) is correct.

(C) $(MN)^T = N^TM^T$

Symmetricity and skew symmetricity depend on nature of M and N, option (C) is incorrect.

(D) adj(MM) = adj(N) adj$M$,

Option (D) is incorrect.

54. A line l passing through the origin is perpendicular to the lines

$l_1 : (3 \quad t)\hat{i} \quad (\ 1 \quad 2t)\hat{j} \quad (4 \quad 2t)\hat{k}, \qquad t$

$l_2 : (3 \quad 2s)\hat{i} \quad (3 \quad 2s)\hat{j} \quad (2 \quad s)\hat{k}, \qquad s$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is (are)

(A) $\frac{7}{3}, \frac{7}{3}, \frac{5}{3}$

(B) (–1, –1, 0)

(C) (1, 1, 1)

(D) $\frac{7}{9}, \frac{7}{9}, \frac{8}{9}$

Answer (B, D)

Hint : $\frac{x}{a} \quad \frac{y}{b} \quad \frac{z}{c}$ (Equation of line l)

Equation of a line l_1, $\frac{x \quad 3}{1} \quad \frac{y \quad 1}{2} \quad \frac{z \quad 4}{2} \quad t$

Equation of a line l_2, $\frac{x \quad 3}{2} \quad \frac{y \quad 3}{2} \quad \frac{z \quad 2}{1} \quad s$

Direction ratio of a line l is given by, $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix}$

$= \quad 2\hat{i} \quad 3\hat{j} \quad 2\hat{k}$

Equation of a line l is $\frac{x}{2}\ \frac{y}{3}\ \frac{z}{2}$

Point of intersection of l and l_1,

$2\quad 3\quad t$ ...(1)

$3\quad 2t\quad 1$ ...(2)

Put the value of t, $3\quad 2(\ 2\quad 3)\ \ 1$

$3\quad 4\quad 6\ \ 1$

$7\quad 7$

1

Point of intersection is (2, –3, 2)

So, $\sqrt{(3\quad 2s\quad 2)^2\quad (3\quad 2s\quad 3)^2\quad (2\quad s\quad 2)^2}\quad \sqrt{17}$

$4s^2\quad 4s\ \ 1\ \ 36\quad 24s\quad 4s^2\quad s^2\quad 17$

$9s^2\quad 28s\quad 20\quad 0$

$s\quad 2,\quad \frac{10}{9}$

i.e., intersection points are (–1, –1, 0) and $\frac{7}{9},\frac{7}{9},\frac{8}{9}$

55. Let $S_n \sum_{k\ 1}^{4n}(\ 1)^{\frac{k(k\ 1)}{2}} k^2$. Then S_n can take value(s)

(A) 1056 (B) 1088

(C) 1120 (D) 1332

Answer (A, D)

Hint : $S_n \sum_{k\ 1}^{4n}(\ 1)^{\frac{k(k\ 1)}{2}} k^2$

$S_n\quad -1^2-2^2\quad 3^2\quad 4^2\quad 5^2\quad 6^2\quad .....\quad (4n\quad 3)^2\quad (4n\quad 2)^2\quad (4n\quad 1)^2\quad (4n)^2$

$S_n = (3^2\quad 1\ )\ \ (4^2\quad 2^2)\ \ (7^2\quad 5^2)\ \ (8^2\quad 6^2)\ \ (11^2\quad 9^2)\ \ (12^2\quad 10^2)$
$.....\ \ (4n\quad 1)^2\quad (4n\quad 3)^2\quad (4n)^2\quad (4n\quad 2)^2$

$S_n = 2(1\quad 3)\quad 2(4\quad 2)\quad 2(7\quad 5)\quad 2(8\quad 6)\quad .....\quad 2(4n\quad 1\quad 4n\quad 3)\quad 2(4n\quad 4n\quad 2)$

$S_n = 2[1\quad 2\quad 3\quad .....\quad 4n]\quad \frac{2\ \ 4n(4n\quad 1)}{2}$

From A, $4n(4n + 1) = 1056$

$4n^2 + n = 264$

$4n^2 + n - 264 = 0$

$n = 8$

From B, $4n(4n + 1) = 1088$ (Not possible)

From C, $4n(4n + 1) = 1120$ (Not possible)

From D, $4n(4n + 1) = 1332$

$n = 9$

SECTION - 3 : (Integer Value Correct Type)

This section contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (*both inclusive*).

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56. Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is , only E_2 occurs is and only E_3 occurs is . Let the probability p that none of events E_1, E_2 or E_3 occurs satisfy the equations $(- 2)p =$ and $(- 3)p = 2$. All the given probabilities are assumed to lie in the interval (0, 1).

Then $\dfrac{\text{Probability of occurrence of } E_1}{\text{Probability of occurrence of } E_3}$

Answer (6)

Hint : $P(E_1)\,P(\bar{E}_2)\,P(\bar{E}_3)$ ...(i)

$P(\bar{E}_1)\,P(E_2)\,P(\bar{E}_3)$ ...(ii)

$P(\bar{E}_1)\,P(\bar{E}_2)\,P(E_3)$ ...(iii)

$P(\bar{E}_1)\,P(\bar{E}_2)\,P(\bar{E}_3)$ ...(iv)

Divide (i) by (iv),

$\dfrac{P(E_1)}{P(\bar{E}_1)}$ _ $\dfrac{\quad}{\quad\quad}$ $\dfrac{\quad 2}{\quad}$

$\dfrac{\quad}{2}$

$\dfrac{P(\bar{E}_1)}{P(E_1)}$ $\dfrac{\quad}{2}$

$\dfrac{1 \quad P(E_1)}{P(E_1)}$ $\dfrac{\quad}{2}$

$\dfrac{1}{P(E_1)}$ 1 $\dfrac{\quad}{2}$

$\dfrac{1}{P(E_1)}$ $\dfrac{\quad}{2}$

$P(E_1)$ $\dfrac{2}{\quad}$ ...(v)

Now, $\frac{\quad}{2}$ $\frac{2}{3}$

$\frac{\quad}{2}$ $\frac{2}{3}$

-3 $= 2$ -4

$= 5$ -4

$= \frac{\quad}{5 \quad 4}$ ...(vi)

Divide (iii) by (iv)

$\frac{P(E_3)}{P(\bar{E}_3)}$ $-$ $\frac{\quad}{\frac{\quad}{2}}$ $\frac{(\quad 2\ )}{\quad}$

$\frac{P(E_3)}{P(\bar{E}_3)}$ $\frac{\quad 2}{5 \quad 4}$

$\frac{P(\bar{E}_3)}{P(E_3)}$ $\frac{5 \quad 4}{\quad 2}$

$\frac{1 \quad P(E_3)}{P(E_3)}$ $\frac{5 \quad 4}{\quad 2}$

$\frac{1}{P(E_3)}$ $\frac{6 \quad 6}{\quad 2}$ $\frac{6(\quad)}{\quad 2}$

$P(E_3) = \frac{2}{6(\quad)}$

$\frac{P(E_1)}{P(E_3)}$ $\frac{\frac{\quad 2}{\quad}}{\frac{\quad 2}{6(\quad)}} = 6$

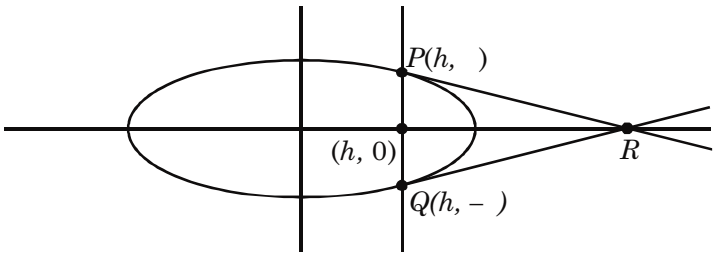
57. A vertical line passing through the point $(h, 0)$ intersects the ellipse $\frac{x^2}{4}$ $\frac{y^2}{3}$ 1 at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If (h) = area of the triangle PQR, $_1 = \max_{1/2 \ h \ 1} (h)$ and $_2 = \min_{1/2 \ h \ 1} (h)$, then $\frac{8}{\sqrt{5}}$ $_1$ 8 $_2$ =

Answer (9)

Hint :

$S \quad \frac{x^2}{4} + \frac{y^2}{3} = 1$

Let P and Q be $(h, \ )$ and $(h, -\ )$



So, R is $\frac{4}{h}, 0$

Now, $= \frac{1}{2} \times 2 \ \times \ \frac{4}{h} - h$

$= \sqrt{3}\sqrt{1 - \frac{h^2}{4}} \times \ \frac{4}{h} - h$

$= \frac{\sqrt{3}}{2} \frac{4 - h^2}{h}^{3/2}$

So, $\frac{d}{dh} < 0$

i.e. is decreasing

i.e. $_1 = \quad \frac{1}{2} \quad = \frac{15\sqrt{45}}{8}$

and $_2 = \ (1) = \frac{9}{2}$

Now $\frac{8}{\sqrt{5}} \ _1 - 8 \ _2 \quad 9$

58. A pack contains n cards numbered from 1 to n. Two consecutive numbered cards are removed from the pack and the sum of the numbers on the remaining cards is 1224. If the smaller of the numbers on the removed cards is k, then $k - 20 =$

Answer (5)

Hint : The smallest value of n for which

$\frac{n(n \ \ 1)}{2} \quad 1224$

$n(n + 1) > 2448$

$n > 49$

For $n = 50$

$\frac{n(n \ \ 1)}{2} \qquad 1275$

So, $k + (k + 1) = 1275 - 1224 = 51$

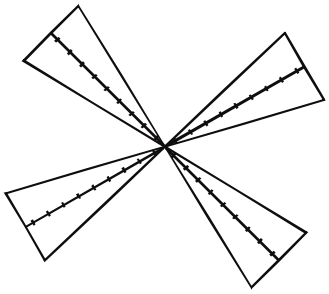
$k = 25$

$k - 20 = 5$

59. Consider the set of eight vectors $V = \{ a\hat{i} \quad b\hat{j} \quad c\hat{k} : a, b, c \quad \{ 1, 1\} \}$. Three non-coplanar vectors can be chosen from V in 2^p ways. Then p is

Answer (5)

Hint : Total 8 vectors are shown in the figure.



Total number of vectors = ${}^8C_3 = 56$

Number of coplaners = $2 \times (6 \times 2) = 24$

$56 - 24 = 32 \quad 2^5$

60. The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then $n =$

Answer (6)

Hint : Let consecutive terms be t_{r+2}, t_{r+1}, t_r

So, $\frac{t_{r\ 1}}{t_r} \quad \frac{10}{5}$

$$\frac{(n \quad 5) \quad (r \quad 1) \quad 1}{r \quad 1} \quad 2$$

$n - 3r + 3 = 0$ ...(i)

also, $\frac{t_{r\ 2}}{t_{r\ 1}} \quad \frac{14}{10}$

$5n - 12r + 6 = 0$ ...(ii)

Solving, $\boxed{n \quad 6}$