Date : 02.06.2013

CODE

4

IIT STUDY CIRCLE

A SHRIRAM GROUP INITIATIVE

Time : 3 hrs. **Max. Marks: 180**

Answers & Solutions

for

JEE (Advanced)-2013

PAPER - 2 (Code - 4)

INSTRUCTIONS

Question Paper Format

The question paper consists of **three parts** (Physics, Chemistry and Mathematics). Each part consists of three sections.

Section 1 contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

Section 2 contains **4 paragraphs** each describing theory, experiment, data etc. **Eight questions** relate to four paragraphs with two questions on each paragraph. Each question of a paragraph has **ONLY ONE correct answer** among the four choices (A), (B), (C) and (D).

Section 3 contains **4 multiple choice questions. Each question has matching lists.** The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

Marking Scheme

For each question in **Section 1,** you will be awarded **3 marks** if you darken **all** the bubble(s) corresponding to only the correct answer(s) and **zero mark** if no bubbles are darkened. In all other cases, **minus one (-1) mark** will be awarded.

For each question in **Section 2 and 3,** you will be awarded **3 marks** if you darken the bubble corresponding to only the correct answer and **zero mark** if no bubbles are darkened. In all other cases, **minus one (-1) mark** will be awarded.

PART–I : PHYSICS

SECTION - 1 : (One or More options Correct Type)

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**.

1. Two vehicles, each moving with speed u on the same horizontal straight road, are approaching each other. Wind blows along the road with velocity w. One of these vehicles blows a whistle of frequency f_1. An observer in the other vehicle hears the frequency of the whistle to be f_2. The speed of sound in still air is V. The correct statement(s) is (are)

   (A) If the wind blows from the observer to the source, $f_2 > f_1$

   (B) If the wind blows from the source to the observer, $f_2 > f_1$

   (C) If the wind blows from observer to the source, $f_2 < f_1$

   (D) If the wind blows from the source to the observer,

Answer (A, B)

Hint : (A) $f_2 = \dfrac{V - w + u}{V - w - u} f_1$ $\qquad (f_2 > f_1)$

(B) $f_2 = \dfrac{V + w + u}{V + w - u} f_1$ $\qquad (f_2 > f_1)$ $\therefore$ Correct.

2. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius $2R$. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are)

   (A) In the region $0 < r < R$, the magnetic field is non-zero

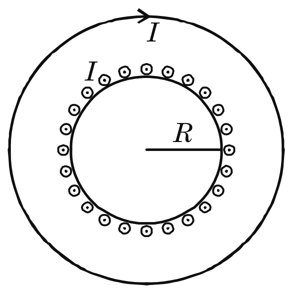
   (B) In the region $R < r < 2R$, the magnetic field is along the common axis

   (C) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centered on the axis

   (D) In the region $r > 2R$, the magnetic field is non-zero

Answer (A, D)

Hint : Assuming B_C for cylinder and B_S for solenoid



(A) $R > r > 0$

$B_S = \mu_0 ni > 0$ (Correct)

(B) $2R > r > R$

$B = \sqrt{B_S^2 + B_C^2}$ not along the axis of cylinder, hence, wrong.

(C) Wrong, not in the plane of circle.

(D) $r > 2R$, $B = B_C$ (Correct)

3. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time $t = 0$ with an initial velocity u_0. When the speed of the particle is $0.5\,u_0$, it collides elastically with a rigid wall. After this collision,

(A) The speed of the particle when it returns to its equilibrium position is u_0

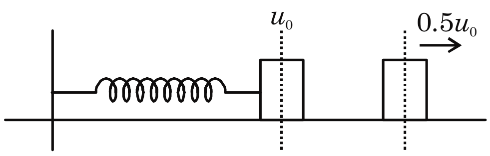
(B) The time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

(C) The time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(D) The time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

Answer (A, D)

Hint :



(A) is correct due to conservation of energy.

(B) $x = A \sin \omega t$

$$\frac{dx}{dt} = A\omega\cos\omega t = \frac{u_0}{2} = u_0 \cos\omega t$$

$$\Rightarrow \quad \cos\omega t = \frac{1}{2}$$

$$\Rightarrow \quad \omega t = \frac{\pi}{3}$$

$$\Rightarrow \quad t = \frac{\pi}{3\omega}$$

$$\Delta t = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{\frac{k}{m}}$$, its wrong.

(C) Maximum compression will occur at time $= \frac{2\pi}{3}\sqrt{\frac{k}{m}} + \frac{\pi}{2}\sqrt{\frac{k}{m}}$

$$= \frac{7\pi}{6}\sqrt{\frac{k}{m}}$$, hence, its wrong.

(D) Second time at equilibrium at $= \frac{2\pi}{3}\sqrt{\frac{k}{m}} + \pi\sqrt{\frac{k}{m}}$

$$= \frac{5\pi}{3}\sqrt{\frac{k}{m}}$$, so correct.

4. Two bodies, each of mass M, are kept fixed with a separation $2L$. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is$G$. The correct statement(s) is (are)

(A) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

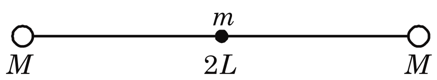
(B) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(C) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(D) The energy of the mass m remains constant

Answer (B)

Hint :



$$0 - \frac{GMm}{L} \cdot 2 + \frac{1}{2}mv^2 = 0$$

$$\Rightarrow \quad v^2 = \frac{4GM}{L}$$

$$\Rightarrow \quad v = \sqrt{\frac{4GM}{L}}$$

$$\Rightarrow \quad v = 2\sqrt{\frac{GM}{L}}$$

5. The radius of the orbit of an electron in a Hydrogen-like atom is $4.5a_0$, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are)

(A) $\frac{9}{32R}$

(B) $\frac{9}{16R}$

(C) $\frac{9}{5R}$

(D) $\frac{4}{3R}$

Answer (A, C)

Hint : $n \cdot \frac{h}{2\pi} = \frac{3h}{2\pi}$

$\Rightarrow$ $n = 3$

$$4.5a_0 = a_0\frac{n^2}{Z}$$

$\Rightarrow$ $Z = 2$

Possible transitions are $3 \rightarrow 2$, $3 \rightarrow 1$ and $2 \rightarrow 1$

For $3 \rightarrow 2$: $\frac{1}{\lambda} = R(2)^2 \left[\frac{1}{4} - \frac{1}{9}\right] = 4R\left[\frac{9-4}{36}\right] = \frac{5R}{9}$

$\Rightarrow \lambda = \frac{9}{5R}$

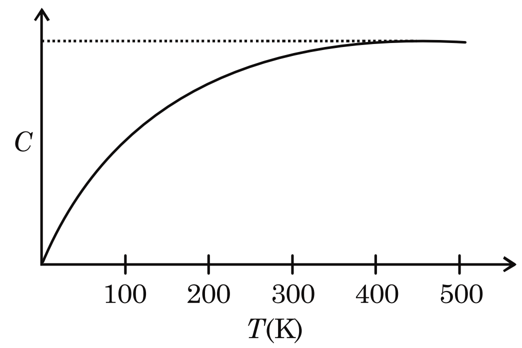
$3 \rightarrow 1$: $\frac{1}{\lambda} = R(2)^2 \left[1 - \frac{1}{9}\right] = 4R\left(\frac{8}{9}\right) = \frac{32R}{9}$

$\Rightarrow \lambda = \frac{9}{32R}$

$2 \rightarrow 1$: $\frac{1}{\lambda} = R(2)^2 \left[1 - \frac{1}{4}\right] = 4R\left(\frac{3}{4}\right) = 3R$

$\Rightarrow \lambda = \frac{1}{3R}$

6. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation.



(A) The rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T.

(B) Heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400-500 K.

(C) There is no change in the rate of heat absorption in the range 400-500 K.

(D) The rate of heat absorption increases in the range 200-300 K.

Answer (B, C, D)

Hint : $dQ = mCdT$

$\Rightarrow \frac{dQ}{dt} = mC$

For $0 < T < 100$ graph is not linear

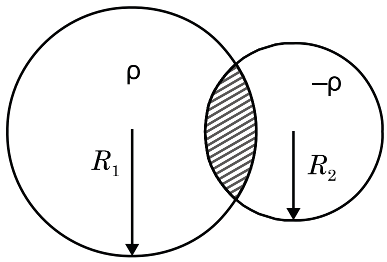
$\Delta Q = m \int CdT$

$= m$ area under C-T graph

For 400-500 K area is more than for 0-100 K

$\frac{dQ}{dt}$ increases for $200 < T < 300$ K as C increases in this region.

7. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region,



(A) The electrostatic field is zero

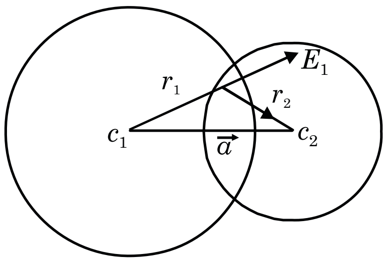
(B) The electrostatic potential is constant

(C) The electrostatic field is constant in magnitude

(D) The electrostatic field has same direction

Answer (C, D)

Hint :



$$\vec{E}_1 = \frac{\rho}{3\varepsilon_0}\vec{r}_1 \qquad \vec{E}_2 = -\frac{\rho}{3\varepsilon_0}\vec{r}_2$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{a}$$

Electric field of constant magnitude and direction.

8. Using the expression $2d \sin \theta = \lambda$, one calculates the values of d by measuring the corresponding angles θ in the range 0 to 90°. The wavelength λ is exactly known and the error in θ is constant for all values of θ. As θ increases from 0°,

(A) The absolute error in d remains constant

(B) The absolute error in d increases

(C) The fractional error in d remains constant

(D) The fractional error in d decreases

Answer (D)

Hint : $d = \frac{\lambda}{2\sin\theta}$

$$|d(d)| = \frac{\lambda}{2}|\text{cosec}\,\theta \cot\theta\, d\theta|$$

Absolute error in d decreases with increase in θ as cosec θ and cot θ decrease with increase in θ.

$$\left|\frac{d(d)}{d}\right| = |\cot\theta\, d\theta|$$

Fractional error decreases with increase in θ.

SECTION - 2 : (Paragraph Type)

This section contains **4 paragraphs** each describing theory, experiment, data etc. **Eight questions** relate to four paragraphs with tow questions on each paragraph. Each question of a paragraph has **only one correct answer** among the four choices (A), (B), (C) and (D).

Paragraph for Questions 9 and 10

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method , a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with power factor unity. All the currents and voltages mentioned are rms values.

9. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(A) 200 : 1 (B) 150 : 1

(C) 100 : 1 (D) 50 : 1

Answer (A)

Hint : $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$\Rightarrow\Rightarrow \frac{V_S}{4000} \quad \frac{10}{1} \qquad V_S \quad 40{,}000$ volt

$\frac{N_P}{N_S} = \frac{V_P}{V_S}$

$= \frac{40{,}000}{200}$

$= 200 : 1$

10. If the direct transmission method with a cable of resistance $0.4\,\Omega\ \text{km}^{-1}$ is used, the power dissipation (in %) during transmission is

(A) 20 (B) 30

(C) 40 (D) 50

Answer (B)

Hint : Total resistance = 0.4 × 20 = 8 Ω

$P = Vi$

$\Rightarrow\ 600 \times 10^3 = 4000 \times i$

$\Rightarrow\ i = \frac{600}{4} = 150$ A

$\therefore\quad P = i^2R$

$= (150)^2 \times 8$

$= 22500 \times 8$

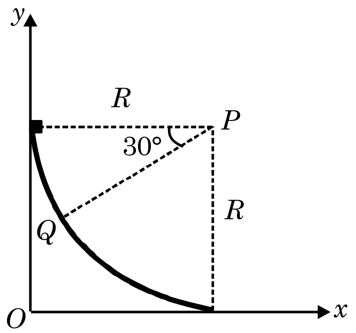
$= 180000$ watt

$= 180$ kW

$\% \text{ loss} = \frac{180}{600} \cdot 100 = 30\%$

Paragraphs for Questions 11 and 12

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J. (Take the acceleration due to gravity, g = 10 ms^{-2})



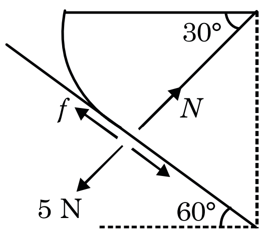
11. The magnitude of the normal reaction that acts on the block at the point Q is

(A) 7.5 N (B) 8.6 N

(C) 11.5 N (D) 22.5 N

Answer (A)

Hint :



$N - 5 = \frac{1 \cdot 100}{40}$

$\Rightarrow \quad N = 7.5$ N

12. The speed of the block when it reaches the point Q is

(A) 5 ms^{-1} (B) 10 ms^{-1}

(C) $10\sqrt{3}$ ms^{-1} (D) 20 ms^{-1}

Answer (B)

Hint : Applying mechanical energy theorem

$-150 = -1 \times 10 \cdot \frac{40}{2} + \frac{1}{2}\ 1\ V^2$

$\Rightarrow \quad V^2 = 100$

$\Rightarrow \quad V = 10$ m/s

Paragraph for Questions 13 and 14

The mass of a nucleus $^{A}_{Z}X$ is less than the sum of the masses of (A-Z) number of neutrons and Z number of protons in the nucleus. The energy equivalent to the corresponding mass difference is known as the binding energy of the nucleus. A heavy nucleus of mass M can break into two light nuclei of masses m_1 and m_2 only if $(m_1 + m_2) < M$. Also two light nuclei of masses m_3 and m_4 can undergo complete fusion and form a heavy nucleus of mass M' only if $(m_3 + m_4) > M'$. The masses of some neutral atoms are given in the table below:

$^{1}_{1}H$	1.007825 u	$^{2}_{1}H$	2.014102 u	$^{3}_{1}H$	3.016050 u	$^{4}_{2}He$	4.002603 u
$^{6}_{3}Li$	6.015123 u	$^{7}_{3}Li$	7.016004 u	$^{70}_{30}Zn$	69.925325 u	$^{82}_{34}Se$	81.916709 u
$^{152}_{64}Gd$	151.919803 u	$^{206}_{82}Pb$	205.974455 u	$^{209}_{83}Bi$	208.980388 u	$^{210}_{84}Po$	209.982876 u

(1 u = 932 MeV/c^2)

13. The kinetic energy (in keV) of the alpha particle, when the nucleus $^{210}_{84}Po$ at rest undergoes alpha decay, is

(A) 5319 (B) 5422

(C) 5707 (D) 5818

Answer (A)

Hint : $^{210}_{84}Po = ^{206}_{82}Pb + ^{4}_{2}He + \Delta E$

$m\left(^{206}_{82}Pb\right) = 205.974455$ u

$m\left(^{4}_{2}He\right) = 4.002603$ u

$m\left(^{206}_{82}Pb\right) + m\left(^{4}_{2}He\right) = 209.977058$ u

$\Delta m = 209.977058 \sim 209.982876$

$= 0.005818$ u

$\Delta E = 0.005818 \times 931.5$

$= 5.419467$ MeV $= 5419.467$ keV

$= 5419.5$ keV

$\boxed{\Delta E > KE_\alpha}$

14. The correct statement is

(A) The nucleus $^{6}_{3}Li$ can emit an alpha particle

(B) The nucleus $^{210}_{84}Po$ can emit a proton

(C) Deuteron and alpha particle can undergo complete fusion

(D) The nuclei $^{70}_{30}Zn$ and $^{82}_{34}Se$ can undergo complete fusion

Answer (C)

Hint : $m\left({}_1^2\text{H}\right)+m\left({}_2^4\text{He}\right)=2.014102+4.002603$

$= 6.016705$ u

$m\left({}_3^6\text{Li}\right) = 6.015123$ u

$m_1 + m_2 > M$

(A is wrong)

$m\left({}_1^1\text{H}\right)+m\left({}_{83}^{209}\text{Bi}\right)=1.007825+208.980388$

$= 209.988213$ u

$m\left({}_{84}^{210}\text{Po}\right)=209.982876$ u

$m_1 + m_2 > M$

(B is wrong)

$m\left({}_1^2\text{H}\right)+m\left({}_2^4\text{He}\right)=2.014102+4.002603$

$= 6.016705$ u

${}_3^6\text{Li} = 6.015123$ u

$(m_3 + m_4) > M'$

Correct (C), deutron and alpha particle can go complete fusion.

$m\left({}_{30}^{70}\text{Zn}\right)+{}_{34}^{82}\text{Se}=69.925325+81.916709$

$= 151.842034$ u

${}_{64}^{152}\text{Gd}=151.919803\text{u}$

$m_3 + m_4 < M'$

(D is wrong)

Paragraph for Questions 15 and 16

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

15. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is

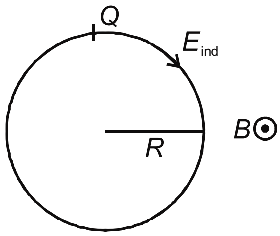
(A) $-\gamma BQR^2$

(B) $-\gamma\frac{BQR^2}{2}$

(C) $\gamma\frac{BQR^2}{2}$

(D) γBQR^2

Answer (B)



Hint : B along (+ Z axis)

$M = \gamma L = \gamma\ m\omega R^2$

(where m = mass of charged particle)

$\boxed{M = \gamma . m\ \omega R^2}$

M will change due to change in ω. Change in ω is given by $|d\omega| = \frac{B\theta}{2m}$

Change in $M = \gamma . m \frac{BQ}{2m} . R^2 = \frac{-\gamma BQR^2}{2}$

Negative sign shows change is opposite to direction of B.

16. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(A) $\frac{BR}{4}$

(B) $\frac{BR}{2}$

(C) BR

(D) $2BR$

Answer (B)

Hint : $\oint \vec{E} . dl = -\frac{d\varphi}{dt} = -A\frac{dB}{dt} \quad \left(\because \frac{dB}{dt} = B \text{ given}\right)$

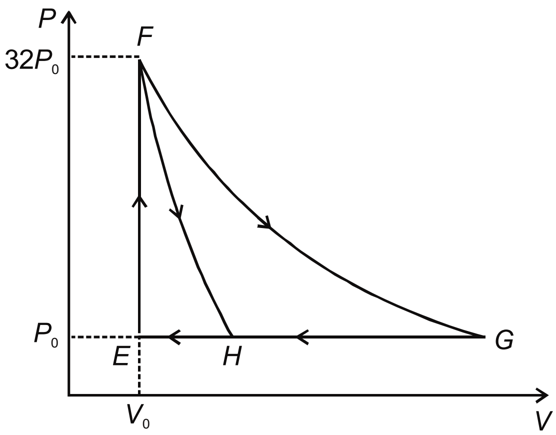
$\Rightarrow \quad |E| = \frac{\pi R^2 . B}{2\pi R}$

$\Rightarrow \quad \boxed{E = \frac{BR}{2}}$

SECTION - 3 : (Matching List Type)

This section contains **4 multiple choice questions. Each question has matching lists.** The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

17. One mole of a monatomic ideal gas is taken along two cyclic processes $E \rightarrow F \rightarrow G \rightarrow E$ and $E \rightarrow F \rightarrow H \rightarrow E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.

	List I		List II
P.	$G \to E$	1.	$160\ P_0V_0 \ln 2$
Q.	$G \to H$	2.	$36\ P_0V_0$
R.	$F \to H$	3.	$24\ P_0V_0$
S.	$F \to G$	4.	$31\ P_0V_0$

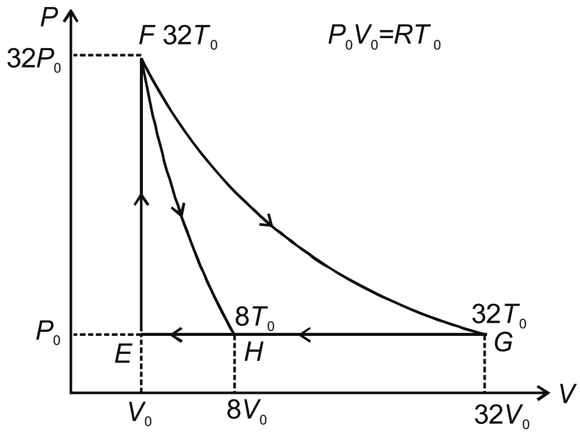
Codes:

	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

Answer (A)

Hint : *FG* is Isothermal

FH is Adiabatic



$$P_0V_G^{\frac{5}{3}} = 32P_0V_0^{\frac{5}{3}} \quad \Bigg| \quad \gamma_{\text{mono}} = 1 + \frac{2}{f} = \frac{5}{3}$$

$$\Rightarrow \quad V_G = (32)^{\frac{3}{5}} V_0 = 8V_0$$

$$\Delta W_{GE} = P_0(V_0 - 32V_0) = -31P_0V_0$$

$P \to 4$

$$\Delta W_{GH} = P_0(8V_0 - 32V_0) = -24P_0V_0$$

$Q \to 3$

$$\Delta W_{FH} = \frac{P_0(8V_0) - 32P_0V_0}{1 - \frac{5}{3}}$$

$$= \frac{-24P_0V_0}{-\frac{2}{3}} = 36P_0V_0$$

$R \to 2$

$$\Delta W_{FG} = 32RT_0 \ln 32$$

$$= 32RT_0 \ln 2^5$$

$$= 160RT_0 \ln 2$$

$S \to 1$

18. Match List I of the nuclear processes with List II containing parent nucleus and one of the end products of each process and then select the correct answer using the codes given below the lists:

List I	List II
P. Alpha decay	1. $^{15}_{8}O \rightarrow ^{15}_{7}N$ .....
Q. β^+ decay	2. $^{238}_{92}U \rightarrow ^{234}_{90}Th$ .....
R. Fission	3. $^{185}_{83}Bi \rightarrow ^{184}_{82}Pb$ .....
S. Proton emission	4. $^{239}_{94}Pu \rightarrow ^{140}_{57}La$ .....

Codes:

	P	Q	R	S
(A)	4	2	1	3
(B)	1	3	2	4
(C)	2	1	4	3
(D)	4	3	2	1

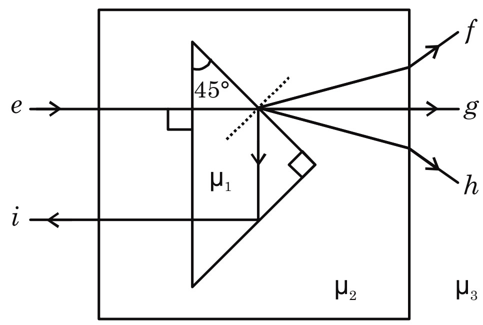
Answer (C)

Hint : In alpha decay, charge number decreases by 2 and mass number decreases by 4.

In B^+ decay, charge number decreases by 1 and mass number remains same.

In proton emission, charge no. decreases by 1 and mass no. decreases by 1.

19. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light *e* enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths '*ef*', '*eg*', '*eh*' or '*ei*'.



Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists:

List I	List II
P. $e \rightarrow f$	1. $\mu_1 > \sqrt{2}\,\mu_2$
Q. $e \rightarrow g$	2. $\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R. $e \rightarrow h$	3. $\mu_1 = \mu_2$
S. $e \rightarrow i$	4. $\mu_2 < \mu_1 < \sqrt{2}\,\mu_2$ and $\mu_2 > \mu_3$

Codes:

	P	Q	R	S
(A)	2	3	1	4
(B)	1	2	4	3
(C)	4	1	2	3
(D)	2	3	4	1

Answer (D)

Hint : For path *e-f*, $\mu_2 > \mu_1$; $\mu_3 < \mu_2$.

(P) → (2)

For path *e-g* $\mu_1 = \mu_2$ (No bending)

(Q) → (3)

For path *e-h*, $\mu_2 < \mu_1$, $\mu_3 < \mu_2$. Also $\mu_1 < \sqrt{2}\,\mu_2$ (No total internal reflection)

(R) → (4)

For path *e-i*, total internal reflection occurs. *i.e.,* $\sin 45° > \frac{\mu_2}{\mu_1} \Rightarrow \mu_1 > \sqrt{2}\mu_2$

(S) → (1)

20. Match List I with List II and select the correct answer using the codes given below the lists:

List I	List II
P. Boltzmann constant	1. $[ML^2T^{-1}]$
Q. Coefficient of viscosity	2. $[ML^{-1}T^{-1}]$
R. Planck constant	3. $[MLT^{-3}K^{-1}]$
S. Thermal conductivity	4. $[ML^2T^{-2}K^{-1}]$

Codes:

	P	Q	R	S
(A)	3	1	2	4
(B)	3	2	1	4
(C)	4	2	1	3
(D)	4	1	2	3

Answer (C)

Hint : $[k] = [ML^2T^{-2}K^{-1}]$

(P) → (4)

$[n] = [ML^{-1}T^{-2}]\ [T]$

$[n] = [ML^{-1}T^{-1}]$

(Q) → (2)

$[h] = [ML^2T^{-2}]\ [T]$

$= [ML^2T^{-1}]$

(R) → (1)

$$[k] = \frac{[ML^2T^{-3}]}{[L][K]}$$

$= [ML^1T^{-3}K^{-1}]$

(S) → (3)

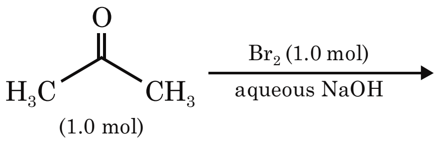
PART–II : CHEMISTRY

SECTION - 1 : (One or More options Correct Type)

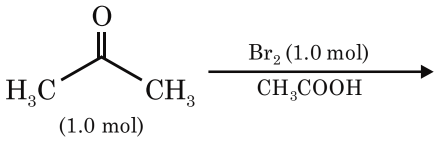
This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**

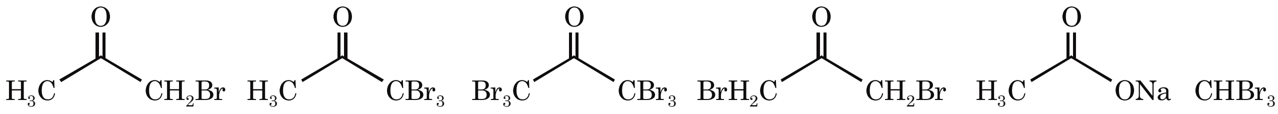
21. After completion of the reactions (I and II), the organic compound(s) in the reaction mixtures is(are)

Reaction I :



Reaction II :



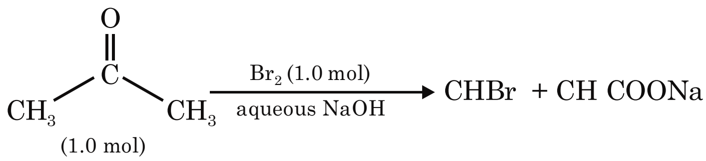


P Q R S T U

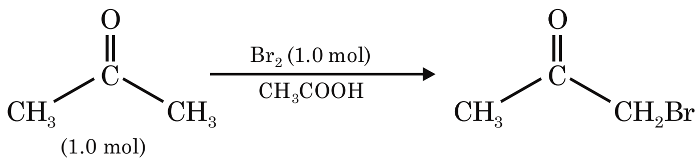
(A) Reaction I : P and Reaction II : P
(B) Reaction I : U, acetone and Reaction II : Q, acetone
(C) Reaction I : T, U, acetone and Reaction II : P
(D) Reaction I : R, acetone and Reaction II : S, acetone

Answer (C)

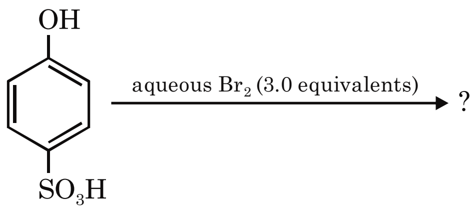
Hint : Reaction I :

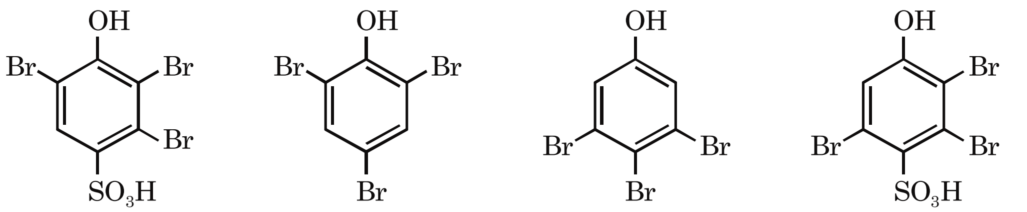


Reaction II :



22. The major product(s) of the following reaction is(are)

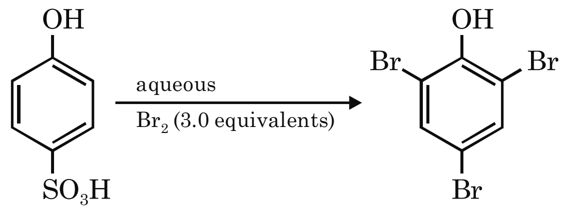




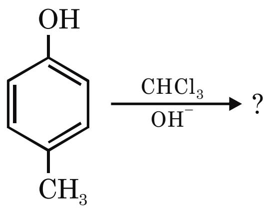
P Q R S

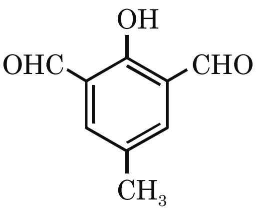
(A) P
(B) Q
(C) R
(D) S

Answer (B)

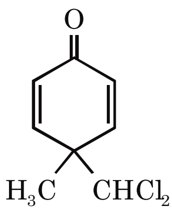
Hint : 

23. In the following reaction, the product(s) formed is(are)

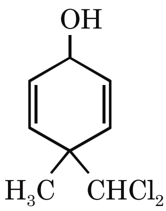




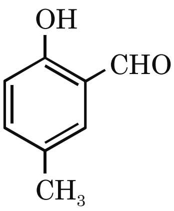
P



Q



R



S

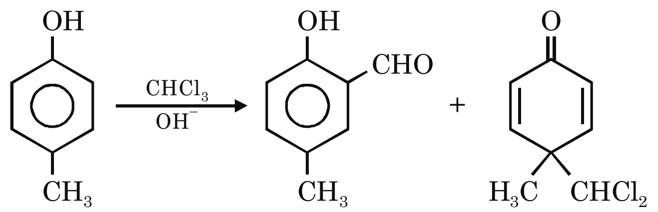
(A) P (major)

(B) Q (minor)

(C) R (minor)

(D) S (major)

Answer (B, D)

Hint : 

(S) (Major product) + (Q) (Minor product)

24. The K_{sp} of Ag_2CrO_4 is 1.1×10^{-12} at 298 K. The solubility (in mol/L) of Ag_2CrO_4 in a 0.1 M $AgNO_3$ solution is

(A) 1.1×10^{-11}

(B) 1.1×10^{-10}

(C) 1.1×10^{-12}

(D) 1.1×10^{-9}

Answer (B)

Hint : The solubility of $Ag_2CrO_4 = [CrO_4^{-2}]$

$$\therefore \quad [CrO_4^{-2}] = \frac{K_{sp}}{[Ag^+]^2} = \frac{1.1 \cdot 10^{-12}}{(0.1)^2} = 1.1 \cdot 10^{-10} \text{ mol L}^{-1}$$

25. The thermal dissociation equilibrium of $CaCO_3(s)$ is studied under different conditions

$$CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$$

For this equilibrium, the correct statement(s) is(are)

(A) ΔH is dependent on T

(B) K is independent of the initial amount of $CaCO_3$

(C) K is dependent on the pressure of CO_2 at a given T

(D) ΔH is independent of catalyst, if any

Answer (A, B, D)

Hint : $CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$

$$K = P_{CO_2}$$

K is only dependent on temperature and it is independent of the amount of reactant or product.

ΔH is dependent on temperature according to Kirchoff's equation but independent of addition of catalyst.

26. The carbon-based reduction method is **NOT** used for the extraction of

(A) Tin from SnO_2

(B) Iron from Fe_2O_3

(C) Aluminium from Al_2O_3

(D) Magnesium from $MgCO_3.CaCO_3$

Answer (C, D)

Hint : Al_2O_3 and $MgCO_3.CaCO_3$ are separately reduced by electrolytic reduction.

27. In the nuclear transmutation

$${}^{9}_{4}Be + X \longrightarrow {}^{8}_{4}Be + Y$$

(X, Y) is(are)

(A) (γ, n)

(B) (p, D)

(C) (n, D)

(D) (γ, p)

Answer (A, B)

Hint : ${}^{9}_{4}Be + \gamma \rightarrow {}^{8}_{4}Be + {}^{1}_{0}n$

$${}^{9}_{4}Be + {}^{1}_{1}H \rightarrow {}^{8}_{4}Be + {}^{2}_{1}H$$

28. The correct statement(s) about O_3 is(are)

(A) O–O bond lengths are equal

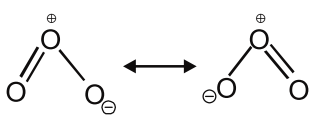
(B) Thermal decomposition of O_3 is endothermic

(C) O_3 is diamagnetic in nature

(D) O_3 has a bent structure

Answer (A, C, D)

Hint :



Ozone is diamagnetic in nature and both the O – O bond length are equal. It has a bent structure.

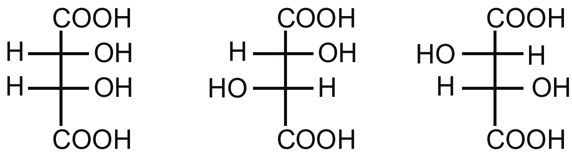
SECTION - 2 : (Paragraph Type)

This section contains **4 Paragraphs** each describing theory, experiment, data etc. **Eight questions** relate to four paragraphs with tow questions on each paragraph. Each question of a paragraph has **only one correct answer** among the four choices (A), (B), (C) and (D).

Paragraph for Questions 29 and 30

P and Q are isomers of dicarboxylic acid $C_4H_4O_4$. Both decolorize Br_2/H_2O. On heating, P forms the cyclic anhydride.

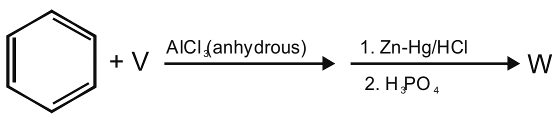
Upon treatment with dilute alkaline $KMnO_4$, P as well as Q could produce one or more than one from S, T and U.

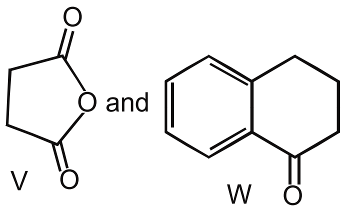


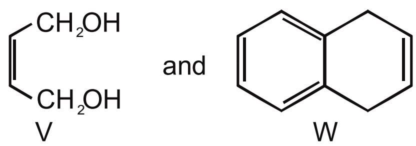
S T U

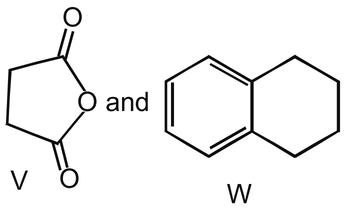
29. In the following reaction sequences V and W are respectively

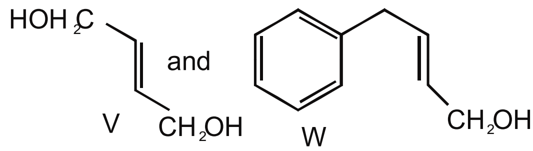
$$Q \xrightarrow[\Delta]{H_2/Ni} V$$



(A) 

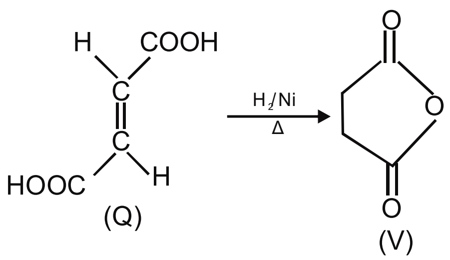
(B) 

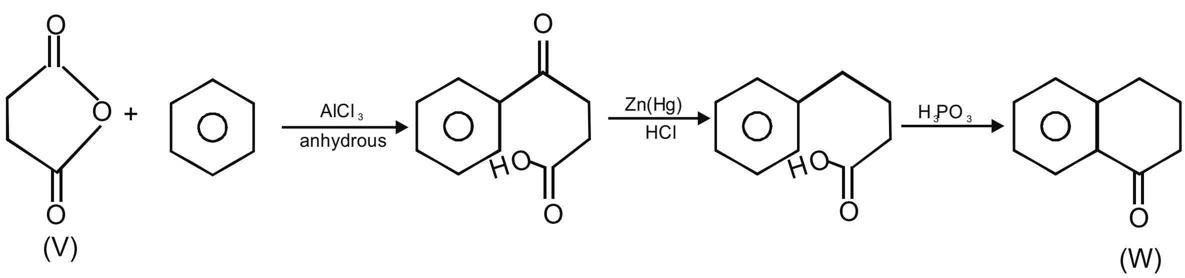
(C) 

(D) 

Answer (A)

Hint :





30. Compounds formed from P and Q are, respectively

(A) Optically active S and optically active pair (T, U)

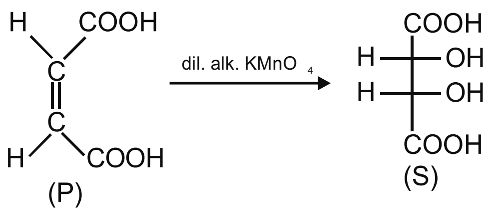
(B) Optically inactive S and optically inactive pair (T, U)

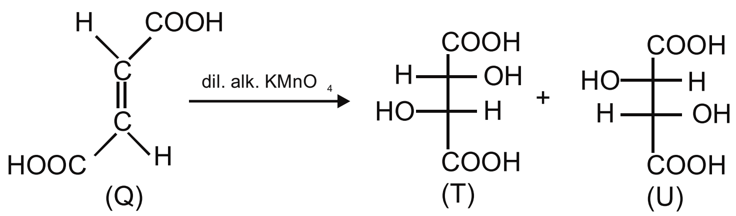
(C) Optically active pair (T, U) and optically active S

(D) Optically inactive pair (T, U)) and optically inactive S

Answer (B)

Hint :





Paragraph for Questions 31 and 32

An aqueous solution of a mixture of two inorganic salts, when treated with dilute HCl, gave a precipitate (P) and a filtrate (Q). The precipitate P was found to dissolve in hot water. The filtrate (Q) remained unchanged, when treated with H_2S in a dilute mineral acid medium. However, it gave a precipitate (R) with H_2S in an ammoniacal medium. The precipitate R gave a coloured solution (S), when treated with H_2O_2 in an aqueous NaOH medium.

31. The coloured solution S contains

(A) $Fe_2(SO_4)_3$ (B) $CuSO_4$

(C) $ZnSO_4$ (D) Na_2CrO_4

Answer (D)

Hint : The filtrate on treatment with ammonical H_2S gives a precipitate which dissolves in aqueous NaOH containing H_2O_2 giving a coloured solution. It contains Cr^{3+} ion.

$$Cr^{+3} + 3NH_4OH \downarrow \underset{\text{(Green)}}{Cr(OH)_3} \quad 3NH_4$$

$$2Cr(OH)_3 + 3H_2O_2 + 4NaOH \rightarrow \underset{\text{(yellow colour)}}{2Na_2CrO_4} + 8H_2O$$

32. The precipitate P contains

(A) Pb^{2+} (B) Hg_2^{2+}

(C) Ag^+ (D) Hg^{2+}

Answer (A)

Hint : Lead salts give white precipitate of $PbCl_2$ with dil. HCl which is soluble in hot water.

Paragraph for Questions 33 and 34

The reactions of Cl_2 gas with cold-dilute and hot-concentrated NaOH in water give sodium salts of two (different) oxoacids of chlorine, P and Q, respectively. The Cl_2 gas reacts with SO_2 gas, in presence of charcoal, to give a product R. R reacts with white phosphorus to give a compound S. On hydrolysis, S gives an oxoacid of phosphorus, T.

33. R, S and T respectively, are

(A) SO_2Cl_2, PCl_5 and H_3PO_4 (B) SO_2Cl_2, PCl_3 and H_3PO_3

(C) $SOCl_2$, PCl_3 and H_3PO_2 (D) $SOCl_2$, PCl_5 and H_3PO_4

Answer (A)

Hint : $SO_2 + Cl_2 \xrightarrow[\text{Catalyst}]{\text{Charcoal}} \underset{(R)}{SO_2Cl_2}$

$\underset{(R)}{10SO_2Cl_2} + P_4 \rightarrow \underset{(S)}{4PCl_5} + 10SO_2$

$\underset{(S)}{PCl_5} + 4H_2O \rightarrow \underset{(T)}{H_3PO_4} + 5HCl$

34. P and Q, respectively, are the sodium salts of

(A) Hypochlorus and chloric acids (B) Hypochlorus and chlorus acids

(C) Chloric and perchloric acids (D) Chloric and hypochlorus acids

Answer (A)

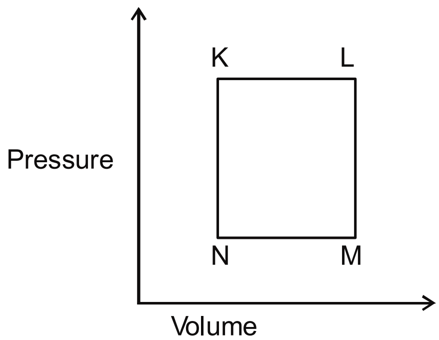
Hint : $Cl_2 + 2NaOH(dil.) \xrightarrow{\text{cold}} \underset{(P)}{NaOCl} + NaCl + H_2O$

$3Cl_2 + 6NaOH(conc.) \xrightarrow{\text{hot}} \underset{(Q)}{NaClO_3} + 5NaCl + 3H_2O$

(P) and (Q) are salts of HOCl and $HClO_3$ respectively.

Paragraph for Questions 35 and 36

A fixed mass 'm' of a gas is subjected to transformation of states from K to L to M to N and back to K as shown in the figure

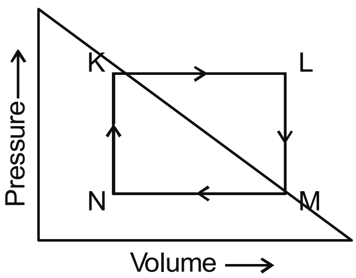


35. The pair of isochoric processes among the transformation of states is

(A) K to L and L to M
(B) L to M and N to K
(C) L to M and M to N
(D) M to N and N to K

Answer (B)

Hint :



As PV product increases temperature increases as PV product decreases temperature decreases.

PV product increases (K → L, N → K)

PV product decreases (M → N, L → M)

Isochoric process are (L → M, N → K)

36. The succeeding operations that enable this transformation of states are

(A) Heating, cooling, heating, cooling
(B) Cooling, heating, cooling, heating
(C) Heating, cooling, cooling, heating
(D) Cooling, heating, heating, cooling

Answer (C)

SECTION - 3 : (Matching List Type)

This section contains **4 multiple choice questions. Each question has matching lists.** The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

---

37. An aqueous solution of X is added slowly to an aqueous solution of Y as shown in List I. The variation in conductivity of these reactions is given in List II. Match List I with List II and select the correct answer using the code given below the lists :

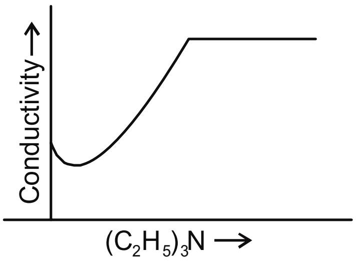
List I		List II
P.	$(C_2H_5)_3N$ (**X**) + CH_3COOH (**Y**)	1. Conductivity decreases and then increases
Q.	KI (0.1M) (**X**) + $AgNO_3$ (0.01M) (**Y**)	2. Conductivity decreases and then does not change much
R.	CH_3COOH (**X**) + KOH (**Y**)	3. Conductivity increases and then does not change much
S.	NaOH (**X**) + HI (**Y**)	4. Conductivity does not change much and then increases

Codes :

	P	Q	R	S
(A)	3	4	2	1
(B)	4	3	2	1
(C)	2	3	4	1
(D)	1	4	3	2

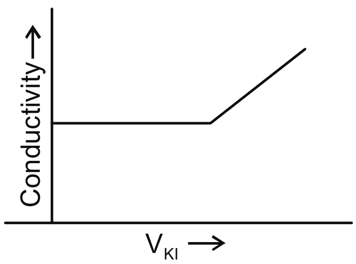
Answer (A)

Hint : (P)



Firstly it decreases due to nutrilization of CH_3COOH and replacement of H^+ by $(C_2H_5)_3\overset{+}{N}H$ but thereafter buffer formation takes place and $[H^+]$ becomes constant and $(C_2H_5)_3\overset{+}{N}H$ increases hence conductivity increases but after equivalence point $(C_2H_5)_3N$ is not ionized due to much higher concentration of $(C_2H_5)_3\overset{+}{N}$ in solution.

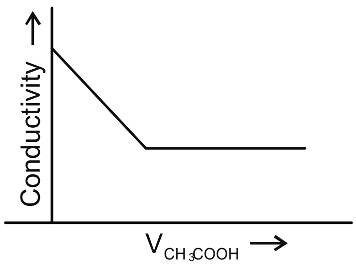
(Q)



$AgNO_3 + KI \longrightarrow AgI(S) \quad KNO_3$

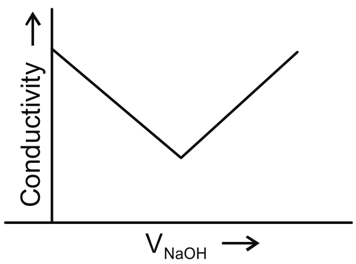
Initially only Ag^+ is replaced by K^+ hence conductivity remain the same thereafter equivalence point $[K^+]$ increases hence conducitity increases.

(R)



Initially conducitity decreases due to replacement of OH^- by CH_3COO^- and then almost constant due to buffer formation

(S)



Decreases due to removal of H^+ by Na^+ then increases due to OH^-.

38. The standard reduction potential data at 25°C is given below.

$E^o(Fe^{3+}, Fe^{2+}) = +0.77$ V;

$E^o(Fe^{2+}, Fe) = -0.44$ V

$E^o(Cu^{2+}, Cu) = +0.34$ V;

$E^o(Cu^{+}, Cu) = +0.52$ V

$E^o[O_2(g) + 4H^+ + 4e^- \rightarrow 2H_2O] = +1.23$ V;

$E^o[O_2(g) + 2H_2O + 4e^- \rightarrow 4OH^-] = +0.40$ V

$E^o(Cr^{3+}, Cr) = -0.74$ V;

$E^o(Cr^{2+}, Cr) = -0.91$ V

Match E° of the redox pair in List I with the values given in List II and select the correct answer using the code given below the lists :

	List I		**List II**
P.	$E^o(Fe^{3+}, Fe)$	1.	–0.18 V
Q.	$E^o(4H_2O \rightleftharpoons 4H^+ + 4OH^-)$	2.	–0.4 V
R.	$E^o(Cu^{2+} + Cu \rightarrow 2Cu^+)$	3.	–0.04 V
S.	$E^o(Cr^{3+}, Cr^{2+})$	4.	–0.83 V

Codes :

	P	Q	R	S
(A)	4	1	2	3
(B)	2	3	4	1
(C)	1	2	3	4
(D)	3	4	1	2

Answer (D)

Hint : (P)

$$Fe^{+3} + e \longrightarrow Fe^{+2} \qquad \Delta^o G_1 = -1F \times 0.77$$

$$Fe^{+2} + 2e \longrightarrow Fe \qquad \Delta^o G_2 = +2F \times 0.77$$

$$Fe^{+3} + 3e \longrightarrow Fe \qquad \Delta^o G_3 = -3F \times E^o_{Fe^{+3}/Fe}$$

$$\Delta^o G_3 = \Delta^o G_1 + \Delta^o G_2$$

$$-3F \times E^o_{Fe^{+3}/Fe} = -0.77\,F + 0.88\,F$$

$$-3E^o_{Fe^{+3}/Fe} = 0.11\,(V)$$

$$E^o_{Fe^{+3}/Fe} = -\frac{0.11\,(V)}{3} = -0.036\,(V)$$

(Q)

$$2H_2O \longrightarrow O_2 + 4H^+ + 4e \qquad E^o = -1.23\text{ V}$$

$$4e + O_2 + 2H_2O \longrightarrow OH^- \qquad E^o = +0.40\text{ V}$$

$$4H_2O \longrightarrow 4H^+ + 4OH^- \qquad E^o = -0.83\text{ V}$$

(R) $Cu^{2+} + 2e \longrightarrow Cu$ $E^o = +0.34$ V

$2Cu \longrightarrow Cu^{2+} + 2eu$ $E^o = -0.52$ V

$Cu^{2+} + Cu \longrightarrow 2Cu^{+}$ $E^o = -0.18$ V

(S) $Cr^{+3} + 3e \longrightarrow Cr$ $\Delta^o G_1 = +3 \times F \times 0.74$

$Cr \longrightarrow Cr^{+2} + 2e$ $\Delta^o G_2 = +2 \times F \times 0.91$

$Cr^{+3} + e \longrightarrow Cr^{+2}$

$E^o_{Cr^{+3}/Cr^{+2}} = -0.4$ V

39. The unbalanced chemical reactions given in List I show missing reagent or condition (?) which are provided in List II. Match List I with List II and select the correct answer using the code given below the lists :

List I	List II
P. $PbO_2 + H_2SO_4 \xrightarrow{?} PbSO_4 + O_2$ + other product	1. NO
Q. $Na_2S_2O_3 + H_2O \xrightarrow{?} NaHSO_4$ + other product	2. I_2
R. $N_2H_4 \xrightarrow{?} N_2$ + other product	3. Warm
S. $XeF_2 \xrightarrow{?} Xe$ + other product	4. Cl_2

Codes :

	P	Q	R	S
(A)	4	2	3	1
(B)	3	2	1	4
(C)	1	4	2	3
(D)	3	4	2	1

Answer (D)

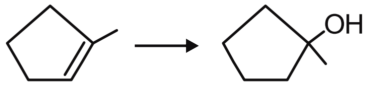
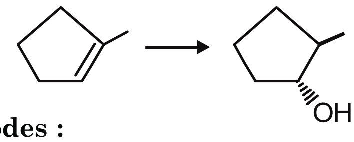
Hint : (P) $2PbO_2 + 2H_2SO_4 \xrightarrow{Warm} 2PbSO_4 + 2H_2O + O_2$

(Q) $Na_2S_2O_3 + 5H_2O + 4Cl_2 \longrightarrow 2NaHSO_4 + 8HCl$

(R) $N_2H_4 + 2I_2 \longrightarrow N_2 + 4HI$

(S) $XeF_2 + 2NO \longrightarrow Xe + 2NOF$

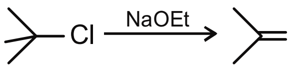
40. Match the chemical conversions in List I with the appropriate reagents in List II and select the correct answer using the code given below the lists :

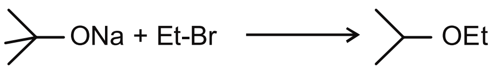
List I	List II
P. 	1. (i) $Hg(OAc)_2$; (ii) $NaBH_4$
Q. 	2. NaOEt
R. 	3. Et-Br
S. 	4. (i) BH_3; (ii) H_2O_2/NaOH

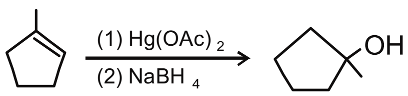
Codes :

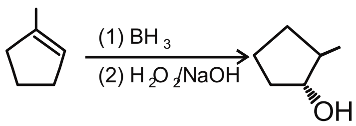
	P	Q	R	S
(A)	2	3	1	4
(B)	3	2	1	4
(C)	2	3	4	1
(D)	3	2	4	1

Answer (A)

Hint : (P) 

(Q) 

(R) 

(S) 

PART–III : MATHEMATICS

SECTION - 1 : (One or More options Correct Type)

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct**

41. In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are)

(A) 16 (B) 18

(C) 24 (D) 22

Answer (B, D)

Hint : QR is largest side

$PM = PN = 2k$

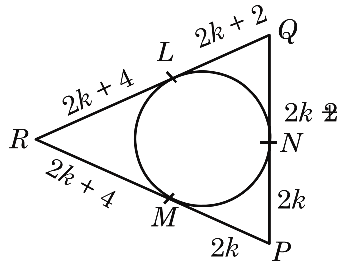
$RM = RL = 2k + 4$

$QL = QN = 2k + 2$

$\therefore$ $QR = 4k + 6$

$RP = 4k + 4$

$QP = 4k + 2$



$$\cos P = \frac{(PQ)^2 + (PR)^2 - (QR)^2}{2(PQ)(PR)}$$

$$\frac{1}{3} = \frac{(4k+2)^2 + (4k+4)^2 - (4k+6)^2}{2(4k+2)(4k+4)}$$

$$\frac{1}{3} = \frac{(2k+1)^2 + 4(k+1)^2 - (2k+3)^2}{4(2k+1)(k+1)}$$

$$\frac{1}{3} = \frac{4(k+1)^2 - 2(4k+4)}{4(2k+1)(k+1)}$$

$$\frac{1}{3} = \frac{(k+1)^2 - 2(k+1)}{(k+1)(2k+1)}$$

$(k + 1)(2k + 1) = 3(k - 1)(k + 1)$

$k = -1$ or $2k + 1 = 3k - 3$

$4 = k$

So,

$PQ = 18$

$QR = 22$

$RP = 20$

42. Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s)

(A) 1 (B) 2

(C) 3 (D) 4

Answer (A, D)

Hint :

$$L_1 : \frac{x-5}{0} = \frac{y}{3-\alpha} = \frac{z}{-2}$$

$$L_2 : \frac{x-\alpha}{0} = \frac{y}{-1} = \frac{z}{2-\alpha}$$

As the two lines are coplanar so $\begin{vmatrix} 5-\alpha & 0 & 0 \\ 0 & 3-\alpha & -2 \\ 0 & -1 & 2-\alpha \end{vmatrix} = 0$

$(5-\alpha)\left[(\alpha-2)(\alpha-3)-2\right]=0$

$(5-\alpha)\left[\alpha^2-5\alpha+6-2\right]=0$

$(5-\alpha)\left[\alpha^2-5\alpha+4\right]=0$

$\boxed{\alpha=5,4,1}$

43. Circle(s) touching x – axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y – axis is (are)

(A) $x^2+y^2-6x+8y+9=0$

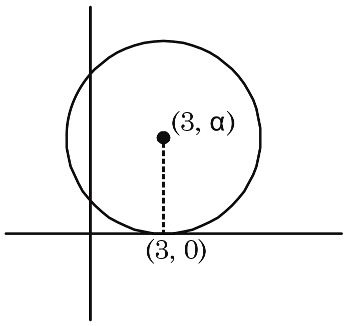
(B) $x^2+y^2-6x+7y+9=0$

(C) $x^2+y^2-6x-8y+9=0$

(D) $x^2+y^2-6x-7y+9=0$

Answer (A, C)

Hint : Clearly centre is $(3, \alpha)$ and radius is $|\alpha|$



So, $x^2+y^2-6x-2\alpha y+c=0$

Radius $9+\alpha^2-c=\alpha^2$

$c=9$

Intercept on y axis, $2\sqrt{\alpha^2-c}=2\sqrt{7}$

$\alpha^2-9=7$

$\alpha=\pm4$

So equation becomes $x^2+y^2-6x\pm8y+9=0$

44. For $a \in \mathbb{R}$ (the set of all real numbers), $a \neq -1$, $\lim\limits_{n\to\infty}\dfrac{(1^a+2^a+\ldots+n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+\ldots+(na+n)]}=\dfrac{1}{60}$.

Then $a =$

(A) 5

(B) 7

(C) $\dfrac{-15}{2}$

(D) $\dfrac{-17}{2}$

Answer (B, D)

Hint : $\underset{n\to\infty}{\text{Lt}}\dfrac{\sum\limits_{r=1}^{n}(r)^a}{(n+1)^{a-1}\left[\sum\limits_{r=1}^{n}(na)+r\right]}=\dfrac{1}{60}$

$\underset{n\to\infty}{\text{Lt}}\dfrac{n^a\sum\left(\dfrac{r}{n}\right)^a}{n(n+1)^{a-1}\sum\left(a+\dfrac{r}{n}\right)}=\dfrac{1}{60}$

$$= \underset{n\to\infty}{\text{Lt}} \frac{1}{\left(1+\frac{1}{n}\right)^{a-1}} \frac{1}{n} \frac{\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{a}}{\sum_{r=1}^{n}\left(a+\frac{r}{n}\right)} = \frac{1}{60}$$

$$= \frac{\int_0^1 x^a}{\int_0^1 a + x} = \frac{1}{60}$$

$$\frac{x^{a+1}\Big|_0^1}{(a+1)\left[\left(ax+\frac{x^2}{2}\right)\Big|_0^1\right]} = \frac{1}{60}$$

$$\frac{1}{a+1}\left[\frac{1-0}{a+\frac{1}{2}}\right] = \frac{1}{60}$$

$$\frac{2}{(2a+1)(a+1)} = \frac{1}{60}$$

$2a^2 + 3a + 1 = 120$

$2a^2 + 3a - 119 = 0$

$$a = \frac{-3 \pm \sqrt{9 + 8(119)}}{4}$$

$$= \frac{-3 \pm \sqrt{961}}{4}$$

$$= \frac{-3 \pm 31}{4}$$

$$\boxed{a = 7, -\frac{17}{2}}$$

45. The function $f(x) = 2|x| + |x + 2| - ||x + 2| - 2|x||$ has a local minimum or a local maximum at $x =$

(A) -2 (B) $\frac{-2}{3}$

(C) 2 (D) $\frac{2}{3}$

Answer (A, B)

Hint :

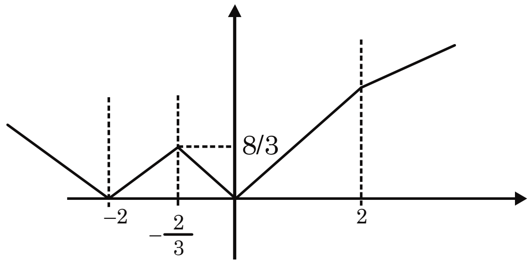
$x < -2,$ $f(x) = -2x - 4$

$-2 \le x < -\frac{2}{3},$ $f(x) = 2x + 4$

$-\frac{2}{3} \leq x \leq 0$, $\qquad f(x) = -4x$

$0 \leq x \leq 2$, $\qquad f(x) = 4x$

$x \geq 2$, $\qquad f(x) = 2x + 4$



Clearly, $x = -2$ and $x = 0$ are point of minima.

$x = -\frac{2}{3}$ is point of maxima.

46. Let ω be a complex cube root of unity with $\omega \neq 1$ and $P = [p_{ij}]$ be a $n \times n$ matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when $n =$

(A) 57 (B) 55

(C) 58 (D) 56

Answer (B, C, D)

Hint : $P = [P_{ij}]_{n \times n}$ $\qquad P^2 \neq 0$

$P_{ij} = \omega^{i+j}$

$$P = \begin{bmatrix} \omega^2 & 1 & \omega & \omega^2 & 1 & \ldots \\ 1 & \omega & \omega^2 & 1 & \omega & \ldots \\ \omega & \omega^2 & 1 & \omega & \omega^2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix}_{n \times n}$$

$$P^2 = \begin{bmatrix} \omega^2 & 1 & \omega & \omega^2 & 1 & \ldots \\ 1 & \omega & \omega^2 & 1 & \omega & \ldots \\ \omega & \omega^2 & 1 & \omega & \omega^2 & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \end{bmatrix} \begin{bmatrix} \omega^2 & 1 & \omega & \ldots \\ 1 & \omega & \omega^2 & \ldots \\ \omega & \omega^2 & 1 & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{bmatrix}$$

$(\omega^4 + 1 + \omega^2) + (\omega^4 + 1 + \omega^2) + \ldots\ldots = 0$

Only when n is a multiple of 3.

$\therefore$ n can be 55, 58, 56 $\quad (P^2 \neq 0)$

47. If $3^x = 4^{x-1}$, then $x =$

(A) $\frac{2\log_3 2}{2\log_3 2 - 1}$ (B) $\frac{2}{2 - \log_2 3}$

(C) $\frac{1}{1 - \log_4 3}$ (D) $\frac{2\log_2 3}{2\log_2 3 - 1}$

Answer (A, B, C)

Hint : $3^x = 4^{x-1}$

$x\log_2 3 = (x-1)\log_2 4$

$x\log_2 3 = (x-1)\ 2$

$2x - x\log_2 3 = 2$

$x\ [2 - \log_2 3] = 2$

$x = \dfrac{2}{2-\log_2 3}$ $\Rightarrow$ (B)

$\Rightarrow \dfrac{2}{2-\dfrac{1}{\log_3 2}} = \dfrac{2\log_3 2}{2\log_3 2 - 1}$ $\Rightarrow$ (A)

$\Rightarrow \dfrac{1}{1-\dfrac{1}{2}\log_2 3} = \dfrac{1}{1-\log_4 3}$ $\Rightarrow$ (C)

So, (A), (B), (C)

48. Let $w = \dfrac{\sqrt{3}+i}{2}$ and $P = \{w^n : n = 1, 2, 3, \ldots\}$. Further $H_1 = \left\{z \in \mathbb{C} : \text{Re}\, z > \dfrac{1}{2}\right\}$ and $H_2 = \left\{z \in \mathbb{C} : \text{Re}\, z < \dfrac{-1}{2}\right\}$, where $\mathbb{C}$ is the set of all complex numbers. If $z_1 \in P \cap H_1$, $z_2 \in P \cap H_2$ and O represents the origin, then $\angle z_1Oz_2 =$

(A) $\dfrac{\pi}{2}$

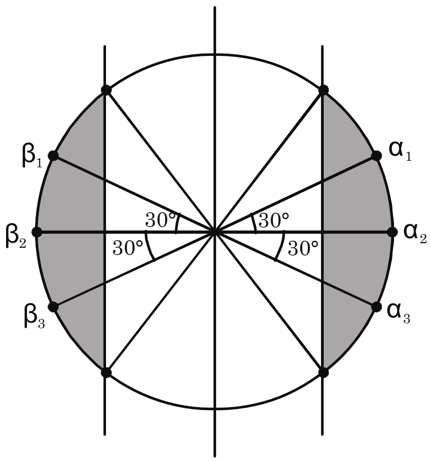
(B) $\dfrac{\pi}{6}$

(C) $\dfrac{2\pi}{3}$

(D) $\dfrac{5\pi}{6}$

Answer (C, D)

Hint : Note that $|\omega| = 1$



α_i are possible value of z_1

β_i are possible value of z_2 i

(= 1,2,3)

$$\omega = \frac{\sqrt{3}}{2} + \frac{i}{2}$$

$$\omega = e^{i\frac{\pi}{6}}$$

$$\omega^2 = e^{i\frac{\pi}{3}}$$

$$\omega^3 = e^{i\frac{\pi}{2}}$$

$$\omega^4 = e^{2i\frac{\pi}{3}}$$

$$\omega^5 = e^{i\frac{5\pi}{6}}$$

So, $\angle z_1 o z_2$ can be $\Rightarrow \frac{2\pi}{3}, \frac{5\pi}{6}$

SECTION - 2 : (Paragraph Type)

This section contains **4 paragraphs** each describing theory, experiment, data etc. **Eight questions** relate to four paragraphs with tow questions on each paragraph. Each question of a paragraph has **only one correct answer** among the four choices (A), (B), (C) and (D).

Paragraph for Questions 49 and 50

Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a$, $a > 0$.

49. If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

(A) $\frac{2}{3}\sqrt{7}$ (B) $\frac{-2}{3}\sqrt{7}$

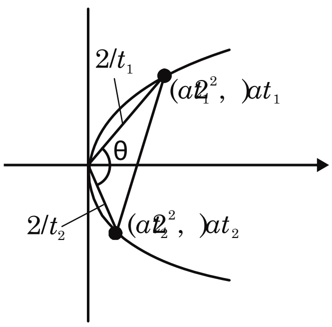
(C) $\frac{2}{3}\sqrt{5}$ (D) $\frac{-2}{3}\sqrt{5}$

Answer (D)

Hint : $t_1 + t_2 = 1$

$t_1 t_2 = -1$

$t_1 = \frac{1 \pm \sqrt{5}}{2}$, $t > 0$



$\therefore$ $t_1 = \frac{1 + \sqrt{5}}{2}$

$$\tan\theta = \frac{\frac{2}{t_1} - \frac{2}{t_2}}{1 + \frac{4}{t_1 t_2}}$$

$$= \frac{2(t_2 - t_1)}{t_1 t_2 + 4} = \frac{2(1 - (1 + \sqrt{5}))}{3} = -\frac{2\sqrt{5}}{3}$$

50. Length of chord PQ is

(A) $7a$ (B) $5a$

(C) $2a$ (D) $3a$

Answer (B)

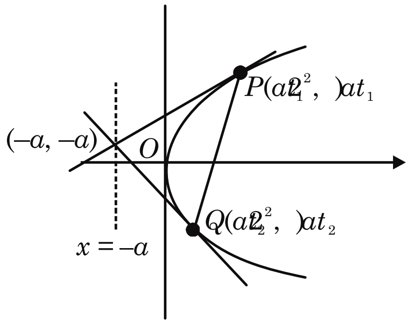
Hint : $a(t_1 + t_2) = a$

$t_1 + t_2 = 1$

$PQ = a(t_1 - t_2)^2$

$= a\left[(t_1 + t_2)^2 - 4t_1t_2\right]$

$= a[1 + 4] = 5a$ *use* (1)



Paragraph for Questions 51 and 52

Let $f : [0, 1] \to \mathbb{R}$ (the set of all real numbers) be a function. Suppose the function f is twice differentiable, $f(0) = f(1) = 0$ and satisfies $f''(x) - 2f'(x) + f(x) \geqslant e^x, x \in [0, 1]$.

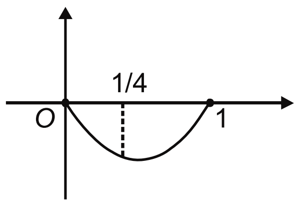
51. If the function $e^{-x} f(x)$ assumes its minimum in the interval [0, 1] at $x = \frac{1}{4}$, which of the following is true?

(A) $f'(x) < f(x), \frac{1}{4} < x < \frac{3}{4}$ (B) $f'(x) > f(x), 0 < x < \frac{1}{4}$

(C) $f'(x) < f(x), 0 < x < \frac{1}{4}$ (D) $f'(x) < f(x), \frac{3}{4} < x < 1$

Answer (C)

Hint :



$\frac{d}{dx}(ye^{-x})$ is an increasing function.

$0 < x < \frac{1}{4}$	$x > \frac{1}{4}$
$\frac{d}{dx}(ye^{-x}) < 0$	$\frac{d}{dx}(ye^{-x}) > 0$
$e^{-x}\frac{dy}{dx} - e^{-x}y < 0$	$e^{-x}\frac{dy}{dx} - e^{-x}y > 0$
$\frac{dy}{dx} < y$	$\frac{dy}{dx} > y$
$f'(x) < f(x)$	$f'(x) > f(x)$

52. Which of the following is true for $0 < x < 1$?

(A) $0 < f(x) < \infty$ (B) $-\frac{1}{2} < f(x) < \frac{1}{2}$

(C) $-\frac{1}{4} < f(x) < 1$ (D) $-\infty < f(x) < 0$

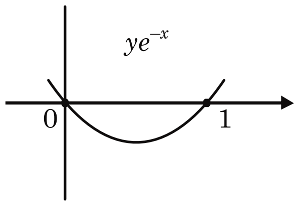
Answer (D)

Hint : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y \geqslant e^x$

$\Rightarrow e^{-x}\frac{d^2y}{dx^2} - 2e^{-x}\frac{dy}{dx} + e^{-x}y \geqslant 1$

$\Rightarrow \frac{d^2}{dx^2}(ye^{-x}) \geqslant 1$

$\Rightarrow ye^{-x}$ is concave up



Hence, $-\infty < f(x) < 0$

Paragraph for Questions 53 and 54

A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

53. If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and the other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$

(B) $\frac{126}{181}$

(C) $\frac{65}{181}$

(D) $\frac{55}{181}$

Answer (D)

Hint : $P\left(\frac{B_2}{WR}\right) = \frac{P\left(\frac{WR}{B_2}\right)P(B_2)}{P\left(\frac{WR}{B_1}\right)P(B_1) + P\left(\frac{WR}{B_2}\right).P(B_2) + P\left(\frac{WR}{B_3}\right).P(B_3)}$

$$= \frac{\frac{{}^2C_1 \cdot {}^3C_1}{{}^9C_2} \cdot \frac{1}{3}}{\left(\frac{{}^1C_1 \cdot {}^3C_1}{{}^9C_2} \cdot \frac{1}{3}\right) + \left(\frac{{}^2C_1 \cdot {}^3C_1}{{}^9C_2} \cdot \frac{1}{3}\right) + \left(\frac{{}^3C_1 \cdot {}^4C_1}{{}^{12}C_2} \cdot \frac{1}{3}\right)}$$

$$= \frac{\frac{2 \cdot 3}{9 \cdot 4}}{\frac{3 \cdot 2}{6 \cdot 5} + \frac{2 \cdot 3}{9 \cdot 4} + \frac{3 \cdot 4 \cdot 2}{12 \cdot 11}}$$

$$= \frac{6 \cdot 5 \cdot 6 \cdot 11}{36 \cdot 181}$$

$$= \frac{55}{181}$$

54. If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$

(C) $\frac{558}{648}$ (D) $\frac{566}{648}$

Answer (A)

Hint :

B_1	B_2	B_3
1W	2W	3W
3R	3R	4R
2B	4B	5B

P(WWW + RRR + BBB)

$$= \left(\frac{1}{6}\cdot\frac{2}{9}\cdot\frac{3}{12}\right)+\left(\frac{3}{6}\cdot\frac{3}{9}\cdot\frac{4}{12}\right)+\left(\frac{2}{6}\cdot\frac{4}{9}\cdot\frac{5}{12}\right)$$

$$= \frac{6+36+40}{648} = \frac{82}{648}$$

Paragraph for Questions 55 and 56

Let $S = S_1 \cap S_2 \cap S_3$ where

$$S_1 = \{z \in \mathbb{C} : |z| < 4\},\ S_2 = \left\{z \in \mathbb{C} : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0\right\} \text{ and } S_3 = \{z \in \mathbb{C} : \text{Re}\, z > 0\}.$$

55. $\min\limits_{z \in S} |1-3i-z| =$

(A) $\frac{2-\sqrt{3}}{2}$ (B) $\frac{2+\sqrt{3}}{2}$

(C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

Answer (C)

Hint : $\min|z-(1-3i)|$

Minimum distance of z from (1, –3)

From question, minimum distance of (1, –3) from $y+\sqrt{3}x=0$ is $\left|\frac{-3+\sqrt{3}}{2}\right| = \frac{3-\sqrt{3}}{2}$

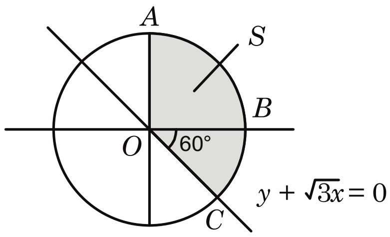
56. Area of S =

(A) $\frac{10\pi}{3}$ (B) $\frac{20\pi}{3}$

(C) $\frac{16\pi}{3}$ (D) $\frac{32\pi}{3}$

Answer (B)

Hint : S_1 represent circle with centre (0, 0) and radius 4



$S_1 : |z| < 4 \Rightarrow x^2 + y^2 < 16$

$$S_2 : \text{Im}\left[\frac{z - 1 + \sqrt{3}i}{1 - \sqrt{3}i}\right] > 0$$

$$\text{Im}\left(\frac{[(x-1) + (y + \sqrt{3}i)][1 + \sqrt{3}i]}{2}\right) > 0$$

$S_2 \equiv y + \sqrt{3}x > 0$

S_3 Re(z) > 0, *i.e.*, $x > 0$

$S = S_1 \cap S_2 \cap S_3$

Area of shaded region is $OAB + OBC = \frac{\pi(4)^2}{4} + \frac{60}{360}\cdot\pi(4)^2$

$$= 4\pi + \frac{16\pi}{6}$$

$$= 4\pi + \frac{8\pi}{3}$$

$$= \frac{20\pi}{3}$$

SECTION - 3 : (Matching List Type)

This section contains **4 multiple choice questions. Each question has matching lists.** The codes for the lists have choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

57. Match List-I with List-II and select the correct answer using the code given below the lists

List-I		**List-II**
P. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 2. Then the volume of the parallelepiped determined by vectors $2(\vec{a}\times\vec{b}), 3(\vec{b}\times\vec{c})$ and $(\vec{c}\times\vec{a})$ is	1.	100
Q. Volume of parallelepiped determined by vectors $\vec{a}, \vec{b}$ and $\vec{c}$ is 5. Then the volume of the parallelepiped determined by vectors $3(\vec{a}+\vec{b}), (\vec{b}+\vec{c})$ and $2(\vec{c}+\vec{a})$ is	2.	30
R. Area of a triangle with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 20. Then the area of the triangle with adjacent sides determined by vectors $(2\vec{a}+3\vec{b})$ and $(\vec{a}-\vec{b})$ is	3.	24
S. Area of a parallelogram with adjacent sides determined by vectors $\vec{a}$ and $\vec{b}$ is 30. Then the area of the parallelogram with adjacent sides determined by vectors $(\vec{a}+\vec{b})$ and $\vec{a}$ is	4.	60

Codes

	P	Q	R	S
(A)	4	2	3	1
(B)	2	3	1	4
(C)	3	4	1	2
(D)	1	4	3	2

Answer (C)

Hint : (P) Given, $[\vec a\ \vec b\ \vec c]=2$

Now, $V = [2(\vec a\times\vec b)\ \ 3(\vec b\times\vec c)\ \ \vec c\times\vec a]$

$= 6[\vec a\ \vec b\ \vec c]^2$

$= 24$

(Q) Given, $[\vec a\ \vec b\ \vec c]=5$

Now, $V = [3(\vec a+\vec b)\ \ \vec b+\vec c\ \ 2(\vec c+\vec a)]$

$= 6[\vec a+\vec b\ \ \vec b+\vec c\ \ \vec c+\vec a]$

$= 12[\vec a\ \vec b\ \vec c]$

$= 60$

(R) Given, $|\vec a\times\vec b| = 40$

Now, $A = \frac{1}{2}|(2\vec a+3\vec b)\times(\vec a-\vec b)|$

$= \frac{1}{2}\cdot 5|\vec a\times\vec b|$

$= \frac{5}{2}\cdot 40$

$= 100$

(S) Given, $|\vec a\times\vec b| = 30$

Now, A $= |(\vec a+\vec b)\times\vec a|$

$= |\vec a+\vec b|$

$= 30$

58. Consider the lines $L_1 : \frac{x-1}{2} = \frac{y}{-1} = \frac{z+3}{1}$, $L_2 : \frac{x-4}{1} = \frac{y+3}{1} = \frac{z+3}{2}$ and the planes $P_1 : 7x+y+2z=3$, $P_2 = 3x+5y-6z=4$. Let $ax+by+cz=d$ be the equation of the plane passing through the point of intersection of lines L_1 and L_2, and perpendicular to planes P_1 and P_2.

Match List-I with List-II and select the correct answer using the code given below the lists:

List-I		List-II	
P.	$a =$	1.	13
Q.	$b =$	2.	–3
R.	$c =$	3.	1
S.	$d =$	4.	–2

Codes :

	P	Q	R	S
(A)	3	2	4	1
(B)	1	3	4	2
(C)	3	2	1	4
(D)	2	4	1	3

Answer (A)

Hint : $L_1 \equiv \dfrac{x-1}{2} = \dfrac{y}{-1} = \dfrac{z+3}{1} = t_1$

and $L_2 \equiv \dfrac{x-4}{1} = \dfrac{y+3}{1} = \dfrac{z+3}{2} = t_2$

For finding point of intersection, we have

$1 + 2t_1 = 4 + t_2$ ...(i)

and $-t_1 = -3 + t_2$ ...(ii)

Solving, we get

$t_1 = 2,\ t_2 = 1$

i.e., point of intersection is (5, –2, –1).

Now, equation of plane P be

$$\begin{vmatrix} x-5 & y+2 & z+1 \\ 7 & 1 & 2 \\ 3 & 5 & -6 \end{vmatrix} = 0$$

$\Rightarrow$ $(-16)(x-5) + 48(y+2) + 32(z+1) = 0$

$\Rightarrow$ $(x-5) - 3(y+2) - 2(z+1) = 0$

$\Rightarrow$ $x - 3y - 2z = 13$

i.e., $a = 1,\ b = -3,\ c = -2,\ d = 13$

59. Match List I with List II and select the correct answer using the code given below the lists :

List I | **List II**

P. $\left[\dfrac{1}{y^2}\left(\dfrac{\cos(\tan^{-1} y) + y\sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)}\right)^2 + y^4\right]^{1/2}$ takes value | 1. $\dfrac{1}{2}\sqrt{\dfrac{5}{3}}$

Q. If $\cos x + \cos y + \cos z = 0 = \sin x + \sin y + \sin z$ then possible value of $\cos\dfrac{x-y}{2}$ is | 2. $\sqrt{2}$

R. If $\cos\left(\dfrac{\pi}{4} - x\right)\cos 2x + \sin x \sin 2x \sec x = \cos x \sin 2x \sec x + 3$. $\cos\left(\dfrac{\pi}{4} + x\right)\cos 2x$ then possible value of $\sec x$ is | $\dfrac{1}{2}$

S. If $\cot\left(\sin^{-1}\sqrt{1-x^2}\right) = \sin\left(\tan^{-1}\left(x\sqrt{6}\right)\right),\ x \neq 0$, then possible value of x is | 4. 1

Codes :

	P	Q	R	S
(A)	4	3	1	2
(B)	4	3	2	1
(C)	3	4	2	1
(D)	3	4	1	2

Answer (B)

Hint :

P. $$\left\{\frac{1}{y^2}\left(\frac{\cos(\tan^{-1} y) + y\sin(\tan^{-1} y)}{\cot(\sin^{-1} y) + \tan(\sin^{-1} y)}\right)^2 + y^4\right\}^{1/2}$$

$$= \left\{\frac{1}{y^2}\left(\frac{\frac{1}{\sqrt{1+y^2}} + \frac{y^2}{\sqrt{1+y^2}}}{\frac{\sqrt{1-y^2}}{y} + \frac{y}{\sqrt{1-y^2}}}\right)^2 + y^4\right\}^{1/2}$$

$$= \left\{\frac{1}{y^2}\left(\frac{y\sqrt{1+y^2}\sqrt{1-y^2}}{1}\right)^2 + y^4\right\}^{1/2}$$

$$= \left\{1 - y^4\right\} + y^4\ ^{1/2} = 1$$

Q. If $\cos x + \cos y + \cos z = \sin x + \sin y + \sin z = 0$

then possible value of $x - y$ is $\pm\frac{2\pi}{3}$ i.e. $\cos\left(\frac{x-y}{2}\right) = \frac{1}{2}$.

R. Given equation can be written is

$$\cos 2x\left\{\cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{\pi}{4} + x\right)\right\} = \sin 2x(1 - \tan x)$$

$$\Rightarrow \cos 2x \cdot 2\sin\frac{\pi}{4}\sin x \quad \sin 2x(1 \quad \tan x) \quad = \quad -$$

$$\Rightarrow \frac{1}{\sqrt{2}}\cos 2x = \cos x - \sin x$$

$$\Rightarrow \frac{1}{\sqrt{2}}(\cos x + \sin x) = 1$$

$$\Rightarrow x = \frac{\pi}{4}$$

So, $\sec x = \sqrt{2}$

S. Given equation is $\frac{x}{\sqrt{1-x^2}} = \frac{\sqrt{6}x}{\sqrt{1+6x^2}}$

either $x = 0$

or $1+6x^2 = 6-6x^2$

$\Rightarrow$ $x = \pm\frac{1}{2}\sqrt{\frac{5}{3}}$

60. A line $L : y = mx + 3$ meets y –axis at $E(0, 3)$ and the arc of the parabola $y^2 = 16x$, $0 \leqslant y \leqslant 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum.

Match List I with List II and select the correct answer using the code given below the lists:

List I	List II
P. $m =$	1. $\frac{1}{2}$
Q. Maximum area of ΔEFG is	2. 4
R. $y_0 =$	3. 2
S. $y_1 =$	4. 1

Codes :

	P	Q	R	S
(A)	4	1	2	3
(B)	3	4	1	2
(C)	1	3	2	4
(D)	1	3	4	2

Answer (A)

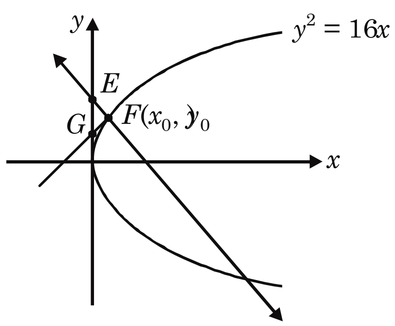
Hint : Parabola is $y^2 = 16x$

and line is $y = mx + 3$

Solving,

$(mx + 3)^2 = 16x$

$\Rightarrow$ $m^2x^2 + (6m - 16)x + 9 = 0$ ...(i)



Also, tangent at F is

$yy_0 = 8(x + x_0)$

So, $y_1 = \frac{8\ \ _0}{y_0\ \ x}$ ...(ii)

Now, area of ΔEFG,

$\Delta = \frac{1}{2}(3 - y_1)\ x_0$

$= \frac{1}{2}\left(3 - \frac{8x_0}{y_0}\right)x_0$

$= \frac{1}{2}\left(3x_0 - \frac{8x_0^2}{y_0}\right)$

$= \frac{1}{2}\left(3x_0 - \frac{8x_0^2}{4\sqrt{x_0}}\right)$

For Δ is to be maximum,

$$\frac{d\Delta}{dx_0} = 0$$

$\Rightarrow \quad 3 - 2 \cdot \frac{3}{2}\sqrt{x_0} = 0$

$\Rightarrow \quad x_0 = 1$

So, $y_0 = 4$

$$y_1 = \frac{8x_0}{y_0} = 2$$

From $y_0 = mx_0 + 3 \quad \Rightarrow \quad m = 1$